CADGME Abstracts

Keynote Addresses

Bruno Buchberger - RISC, University of Linz, Austria

Title | **Lazy Thinking: A Reasoning Strategy for Inventing Algorithms**

Abstract

In this talk, we will first summarize:

- the fundamental role of mathematics within science and technology,
- the fundamental role of reasoning within mathematics,
- and the importance of teaching the art of reasoning for mathematical education.

In the main part of the talk, we will present a general reasoning strategy, called "Lazy Thinking", for the systematic invention of algorithms. The strategy can be used both for guiding humans and for guiding automated reasoning systems in the invention algorithms.
The Lazy Thinking strategy combines two natural ideas in a systematic way:
Given a specification S of a problem,

- apply an algorithm scheme A (condensed knowledge on basic principles for
- constructing an algorithm A from subalgorithms AA)
- try to prove that the algorithm scheme solves the problem; this attempt will normally fail because the subalgorithms AA occurring in the scheme are unspecified; from the failing proof derive specifications SS for the subalgorithms AA.

Now, either we already have algorithms AA that satisfy the specifications SS in our algorithm library or we apply the Lazy Thinking strategy recursively to SS in order to invent suitable subalgorithms AA.
We will illustrate the power of Lazy Thinking, first, by a simple example (sorting) and then by a non-trivial example (the construction of Groebner bases, which is known to be a fundamental and difficult problem in polynomial algebra). Some thoughts on the future of mathematics in the light of increasing sophistication and automation of mathematical reasoning will be presented in the conclusion of the talk.

Vlasta Kokol-Voljc - University of Maribor, Slovenia

Title | **Assessment in CAS and DGS Environment**

Abstract

Changes in the teaching methods and in teaching focuses, caused by the use of Computer Algebra Systems and Dynamic Geometry Software in mathematics teaching inevitably influence also the assessment. Using CAS and DGS requires refocusing of the exams and exam questions.

- What has to be changed in a new teaching environment?
- How do the new circumstances influence assessment?
- What remains to be asked from mathematical point of view?

Questions like these are asked by the teachers who are courageous enough to use technology like CAS or DGS in their mathematics classes. Let us think about this together!

Colette Laborde - Universite Joseph Fourier, Grenoble, France

Title **The design of tasks taking full advantage of dynamic geometry: what kinds of knowledge does it require from teachers?**

Abstract

Bridging the gap between the potentialities of dynamic geometry and the reality of mathematics teaching is a critical issue in current research. Although dynamic geometry is very promising in terms of learning potentialities, it appears that often the usual teaching practice does not take full advantage of these possibilities. The talk will give a short overview of ordinary uses of dynamic geometry in the schoolbooks or in the classrooms as they have been investigated by research. Then it will analyze the complexity of the process of designing tasks based on dynamic geometry and show how such a process involves the intertwining of different kinds of knowledge.

Kenneth Ruthven – University of Cambridge, UK

Title **Herschel's heritage and today's technology integration: a postulated parallel**

Abstract

In 1833, a scientific paper by the astronomer-mathematician Herschel recommended a new technology to his fellow researchers. As interest in this technology grew amongst scientists and engineers, it was commercialised, and competing manufacturers entered the market. Over the latter half of the nineteenth century its price fell by roughly two orders of magnitude and its uptake increased enormously. By the early twentieth century, enthusiasm for this technology had spread to schools. It became closely associated with a wider reform movement around 'heuristic methods' which laid emphasis on giving pupils 'practical experience' and encouraging their 'self activity', sometimes in specially-equipped 'mathematical laboratories'. Advocacy for such reforms reached a peak in 1912 at a study conference organised in Cambridge by the International Commission on the Teaching of Mathematics. Although the eventual impact of the movement as a whole was limited, Herschel's technology successfully worked its way into mainstream curriculum and classroom.
A century later, does this precedent provide a parallel to today's attempts to integrate CAS and DGS technologies into school mathematics?

Edith Schneider - University of Klagenfurt, Austria

Title **CAS as didactical challenge**

Abstract

Since approx. 20 years Computer Algebra Systems (CAS) are used in mathematics classrooms all over the world and different aspects of their influences and effects are didactically analysed and investigated. However, the fact is dealt extremely reserved (almost bashful) that CAS nearly perfectly master all for what is spent most of the time in mathematics classes, namely numerical and algebraic manipulations. This fact poses a great and important didactical challenge for the teaching of mathematics, but it also offers interesting possibilities. In the lecture such didactical changes and potentials should be presented and discussed with concrete examples (also with the new TI-Nspire CAS).

Luc Trouche - National Institute of Pedagogical Research, Lyon, France

Title	**Resources for teaching mathematics in ICT environments: which, who and how?**

Abstract

Designing pedagogical resources well fitted to teach mathematics in various ICT environments is far from being obvious. Some questions often arise:

- what kind of numerical resources (is there a best model, or necessary components of a model, or necessary qualities.)
- who is to be involved in the process of resources design (a single role, or a set of roles: designer-author-engineer-user; team, network or community of practice)
- how could the process of design be managed (relationships between design and usages, between pioneers and ordinary classrooms.).

We will propose some elements of answers based on recent experiments and point out a few new questions.

Contributed Talks

Name	**G. Aguilera, J.L. Galán, Y. Padilla, and P. Rodríguez**
Affiliation	University of Malaga, Spain
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Title	**Developing Propositional Classical Logic Automated Theorem Provers for Education**
Working Group	Automated Reasoning and Mathematical Education

Abstract

The use of Computer Algebra Systems (CAS) in mathematical education is widely used as a practical tool. However CAS are not used frequently for teaching topics as automated reasoning.
This work is focused on the developing of a package called ATPCL. The package has been developed using DERIVE, but can be rewritten in other CAS. ATPCL includes some methods for automated reasoning in Propositional Classical Logic, "Quine" and "Semantic Tableaux". Another auxiliary package called TREE to deal with tree structure has been also developed.
The utility file has been developed for helping teacher and students involved in automated reasoning. Specifically, the immediate goal was the students of the subject "Computational Logic" in Computer Science Degree in Malaga University.
The package allows teachers to show examples of testing the validity or satisfiability of formulae and to verify deductions of a formula from a set of formulae in Propositional Classical Logic using Quine's method or Semantic Tableaux. Students can also use this utility file for testing their own exercises and even for learning how to do them.
Our group has developed other packages for helping in mathematical education. Some of them are: Line Integrals, Surface Integrals, Multiple Integrals, Complex Variable, Random Variables and Graph theory. These packages have become a useful tool in the teaching-learning process in Mathematics.
Nowadays, our group is developing other packages for helping teachers of Mathematics, and has other projects as the development of a package for automated reasoning in First Order Logic and another one to deal with the main topics of Discrete Mathematics.

Name	**Yimaz Aksoy, Mehmet Bulut, and Seref Mirasyedioglu**
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Title	**The effect of Computer Algebra System (CAS) in the development of conceptual and procedural knowledge**
Working Group	Relating procedural and conceptual knowledge of mathematics through CAL

Abstract

This report explores the effect of CAS in the development of procedural and conceptual knowledge of first year undergraduate mathematics and science education students. MAPLE was used as CAS in the teaching of Calculus concepts. As derivative is used mainly by mathematics and science education lessons, we chose this concept for comparing the development of procedural and conceptual knowledge of first year undergraduate mathematics and science education students. The students in both groups were encountered with the derivative concept for the first time. In both of the groups, students have been studied as groups of 2 or 3 students. The lessons were taken in the laboratory and the students had the opportunity to use laboratory besides the lessons. In order to teach the derivative concept, student cantered activities have been designed. While designing these activities, guides were given to students to use the MAPLE. Teaching the derivative concept has been designed as two consecutive steps: At first step; students studied on the concept of derivative as rate of change. At this step real life problems about rate of change have been given to students. By solving these problems students have discovered the concept of the derivative. Students interpreted graphics of functions for developing conceptual understanding of rate of change. At second step, activities have been designed by including geometrical, numerical and symbolic (algebraic) representations of the derivative concept. As it is indicated in many researches, according to quantitative results of the study in both groups students develop their procedural skills as well as their conceptual knowledge. Quantitative results suggest that mathematics education students scored higher on calculus-computational and conceptual-knowledge exam. Also the results showed that mathematics education students develop a tendency for tangent-oriented of the derivative aspects while science education students develop a tendency of the derivative aspects.

Name	**Muharrem Aktümen and Tolga Kabaca**
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Title	**Constructing the disk method formula the volume obtained by obtaining revolved curve around an axis by CAS**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

Calculus concepts should have been taught in a carefully designed learning environment, because these concepts constitute a very important base for almost all applied sciences. Integral, one of the fundamental concepts of Calculus, has a wide application area. In the web site of the Turkish Language Association, the definitions of the concept of integral "The total consists of pieces" and "(Mathematics) function that, its derivative is known" are given. First meaning includes finding the areas that under curves, calculating the volume of various material things and other application areas that is named as definite integral. This paper focuses on that constructing the disk method formula for the volume obtained by revolution a curve around an axis by the help of CAS. In this study, a semi-structured interview was carried out. In this interview, it is tried to construct the disk method formula. Levels of constructing the disk method formula in this study: • Entrance to the concept: evaluate the volume of an Egypt pyramid. • Evaluate the volume of a cone and any object obtained by revolution (By Maple worksheet) • Designing their own rings and evaluating its price (By Maplet). You can find the Maplet which designing their own rings, prepared in Maple 9 from the address of http://w3.gazi.edu.tr/web/aktumen/diskmethod/disk.htm In this study, the interview has been presented as a dialog between teacher and students. When we look at feedback of students, it is seen that such a teaching method effects students in a positive way and causes to gain conceptual understanding directed towards the concepts of approximations and volume.

Name	**Tünde Berta**
Affiliation	Eötvös Loránd University, Budapest, Hungary
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Title	**Graphical Calculators in Pre- and In-Service Teacher Training**

Abstract

The role and significance of graphic calculators have been in the focus of continuous discussion and debate since their appearance in education, particularly in mathematics education. For that reason, we hold it an important element of teacher training to prepare them how to handle these problems, and how to integrate newer and newer technology in their practice. In the present talk, I describe a few didactical aspects applied in my lecture and some experiences among my students.

Name	**Roger Brown, Einir Wyn Davies, Peter Flynn, and John Monaghan**
Affiliation	University of Leeds, UK
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Title	**The introduction of symbolic calculators into assessment in mathematics at the International Baccalaureate Organisation: aspects of item design and student performance**
Working Group	General Session - Other Topics

Abstract

The International Baccalaureate Organisation (IBO) has run pilot Higher Level examinations in which candidates are allowed to use a symbolic calculator (the TI-89). This talk/paper reports on the background of this development and aspects of item design and student performance. Background issues include brief details of IBO Mathematics curricula and assessment, aspirations and concerns for a 'computer algebra allowed' examination and issues in establishing an equitable pilot examination. Item design issues considered include amending (or not) existing Higher Level Mathematics questions and the skills students are expected to exhibit in their answers. Student performance examines overall student performance and student performance on selected questions in the IBO Higher Level Paper 1 of May 2006. The talk/paper ends with a discussion of issues raised.

Name	**Natalia Budinski**
Affiliation	University of Novi Sad, Serbia
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Title	**Computer generated questionnaires to evaluate students' understanding of mathematical concepts**
Working Group	General Session

Abstract

This paper shows how to construct the questionnaire based on the basic methodical concepts with computer. The methods of generation uniformly tasks are analyzed and done in Windows platform in Word format. This paper shows computer applications generating a questionnaire each for every student. The questionnaire is consisted of five to seven different quadratic equation tasks and attempts to measure students' understanding of mathematical concepts.

Name	**Mehmet Bulut, Yýlmaz Aksoy, and Beref Mirasyedioglu**
Affiliation	Gazi University, Turkey
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Title	**The effects of CAS in the development of problem solving abilities**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

This study investigates the effect of CAS in the development of problem solving abilities of first year undergraduate mathematics and science education students. MAPLE was used as CAS in the teaching of Calculus concepts. As derivative is used mainly by mathematics and science education lessons, we chose this concept for comparing the development of problem solving abilities of first year undergraduate mathematics and science education students. The students in both groups were encountered with applications of derivative for the first time. Cooperative learning in constructivist environment was implemented in both of the experimental and control groups. Differently, in experimental group, the lessons were taken in the computer laboratory and the students had the opportunity to use laboratory besides the lessons. In order to teach applications of derivative, student cantered activities have been designed. While designing these activities, guides were given to students to use the MAPLE. Teaching the applications of derivative has been designed in interdisciplinary context. For this, real life problems about applications of derivative have been given to students. By solving these problems students have discovered and applied the concept of the derivative. According to conclusions of many research, problem solving abilities of students improved by using CAS. In this study, it is found that mathematics education students scored higher on calculus-problem solving exam. According to qualitative and quantitative results of the study students in experimental group scored higher in this exam. Also the results showed that mathematics education students developed more problem solving abilities than science education students.

Name	**Jack Carter and Beverly J. Ferrucci**
Affiliation	California State University, USA
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Title	**Compositions of Dilations and Isometries in Calculator-Based Dynamic Geometry**
Working Group	Research perspectives of the impact of dynamic mathematics on teaching and learning

Abstract

In an exploratory study pre-service elementary school teachers constructed dilations and isomerises for figures drawn and transformed using dynamic geometry on calculators. Initially, students constructed images using a single transformation, either a reflection, rotation, translation, or dilation. Students next used the technology to make images that were the result of an isometric and a size transformation. Video analysis of students' constructions of the transformational images revealed that most difficulties related to misconceptions about basic geometric measures and to inexperience in adjusting or revising the defining aspects that determined the images under the transformations. Observational and self assessments of the constructed images showed that the future teachers developed high levels of confidence in their abilities to construct compositions of the geometric transformations. Follow-up assessment items required students to find both pre-images and images of the isomerises and dilations as well as geometric features that described the transformations. Scores on these items indicated that the prospective teachers developed levels of expertise with the compositions of transformations that corresponded to their levels of confidence. A review of the instructional materials and video clips of the instructional episodes illustrated the pre-service teachers' successful constructions and common obstacles that they encountered. Conclusions indicated that dynamic geometry on the calculator is an example of the application of appropriate technology for developing these future teachers' expertise with compositions of geometric transformations.

Name	**Yip-Cheung Chan**
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Title	**Different approaches of interplay between experimentation and theoretical consideration in dynamic geometry exploration: An example from exploring Simon line**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

Dynamic geometry environment (DGE) is a powerful tool for exploration and discovering geometric properties because it allows users to (virtually) manipulate geometric objects. There are two possible functions of DGE, viz. experimentation and theoretical consideration. Different people may use DGE differently. On one extreme, the user may tries to discover the geometric properties only by trial-and-error strategies, without any theoretical consideration. On another extreme, the user may consider the problem by mathematical theory and uses DGE merely for the purpose of "externalize" his/her previous ideas. However, it seems that the approaches in DGE explorations are not in dichotomy but rather a spectrum from experimentation approach to theoretical approach. In other words, in most cases, there is interplay between experimentation and theoretical consideration. Depending on the specific mathematical tasks and the background of individual users, some approaches are more experimentation whereas some other approaches are more theoretical. In this talk, different approaches of exploring a geometric task using Sketchpad (a kind of DGE) by four participants who have strong mathematics background will be compared. The task aimed at discovering the defining property of Simon line. In this study, two very different approaches of exploration were observed. Among the four participants, approaches of two participants were relatively more experimental oriented whereas approaches of another two participants were relatively more theoretical oriented. Interplay between experimentation and theoretical consideration were observed in their exploration processes. Moreover, two participants have invented some new methods of using Sketchpad to overcome some possible limitations of the software. The result suggests that DGE is not necessary a purely experimentation platform but can also be a platform that may led to interplay between experimentation and theoretical consideration.

Name	**Adrian Craciun, Madalina Hodorog**
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Title	**The Quotient–Remainder Theorem for Natural Numbers: Discovery by Lazy Thinking**
Working Group	Automated Reasoning and Mathematical Education

Abstract

We present part of a case study in systematic theory exploration of natural numbers, using a scheme based exploration model recently proposed by Bruno Buchberger: the discovery of the quotient-remainder decomposition
of natural numbers. We start from the well-known axioms of natural numbers and develop the theory in exploration rounds, by introducing new notions (using definition schemes), exploring their properties (using proposition schemes), solving problems involving the notions (introduced by problem schemes). The particular problem we focus on is: for two natural numbers, x, y find another, z, such that x = yz (x can be decomposed in the product of y and z). To solve the problem we apply lazy thinking, i.e. we try to invent an algorithm A to solve the problem x = yA[x, y], by proposing an algorithmic idea (scheme) for A, and proving that it solves the problem. As expected, this problem cannot be solved, and we show how the analysis of the failure leads to the quotient-remainder decomposition of natural numbers, i.e. for any natural numbers x, y,y positive, x = yq[x, y]+r[x, y]. By lazy thinking, we discover the appropriate algorithms q and r. This case study reflects the pedagogical value of the exploration model and of the lazy thinking method in particular, especially when supported by a system like Theorema (used in our case study), that allows natural style (close to textbook mathematics) input and output. We emphasize the role of using accumulated mathematical experience (through knowledge schemes), learning from failure, expanding the available inference machinery (adding new inference rules), and experimentation. Work supported by EU Marie Curie Project MERG-CT-2004-01271

Name	**Peter Csiba**
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Title	**Application of mathematical software's on J. Selye University in Komarno**
Working Group	General Session - Other Topics

Abstract

On J. Selye University we applied in educational process free mathematical software's like Winplot, Wingeom, Winstats, Geogebra and computer algebra system MuPAD Pro. The members of mathematical departments participate in localization of these free projects. Winplot is a general-purpose plotting utility, which can draw (and animate) curves and surfaces presented in a variety of formats. Wingeom is for high-precision geometric constructions in both two and three dimensions. Winstats provides access to scatter plots, curve fitting, histograms, statistical data, and standard theoretical probability distributions. It also simulates dealing cards, sampling candy, tossing darts, needles and coins. GeoGebra is mathematical software that combines dynamic geometry, algebra and calculus. Teaching of mathematics was realized on two faculties with different profiles of students. On the Pedagogical Faculty in the teacher-training we used all this software's on lectures or seminars, but the students may deal minutely with this computer programs on facultative courses. On the Faculty of Economics students have a facultative course, where they deal with application of software Winplot and CAS MuPAD in own study and research.

Name	**Dominik Dietrich and Mark Buckley**
Affiliation	Saarland University, Germany
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Title	**Verification of Human-level Proof Steps in Mathematics Education**
Working Group	Automated Reasoning and Math Education

Abstract

The importance of training students in theorem proving skills is becoming increasingly recognised and implemented in second-level curricula. It has been shown that tutors use their evaluations of the correctness of proof steps to generate pedagogically suitable feedback. Hence it is necessary for an ITS to have a reasoning component supporting this service.
Automated theorem proving is one of the key research areas of artificial intelligence, and powerful mechanisms have been developed. However, tutoring systems for mathematics usually use special purpose domain reasoners, ignoring the progress made in the field so far. One reason for this might be that there is a large gap between the formal reasoning style implemented by traditional theorem provers and the informal reasoning patterns of students. In contrast to this, proof planning offers a sufficiently high level of reasoning and thus minimises this gap.
We argue that proof planning is a suitable framework for tutoring mathematics. We show how the existing techniques implemented in the mathematical assistant OMEGA are extended to verify proof steps given by a student. We are able to determine and to maintain multiple possible interpretations consistent with the proof step, which can occur due to the ambiguity and underspecification introduced by, for instance, incomplete descriptions of steps. The result of our analysis is an answer whether the step is correct or not, and if correct, the information justifying the student's step, which can be used for further analysis, such as relevance and granularity of the step.
Our approach allows us to dynamically model the proof that the student is building. In particular, solutions need not be predefined to determine correctness. Thus the student is free to build any valid proof of the theorem at hand, and proof steps are always analysed in the context of the partial proof so far.

Name	**Andreas Fest, Ulrich Kortenkamp, Anne Geschke, and Dirk Materlik**
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Title	**Teaching Graph Algorithms with Visage**
Working Group	Future Trends in Interactive Geometry

Abstract

The current Berlin curriculum contains new topics on discrete mathematics and graph algorithms. Two modules on "discrete structures" and "Optimal paths" are proposed to be taught in mathematics courses at secondary school. Teachers are encouraged to use adequate teaching software. The presented software package "Visage" is an extension of the interactive geometry system "Cinderella" that can be used for visualization of graph algorithms. Using the intuitive user interface of Cinderella, graphs and networks can be drawn very easily and different textbook algorithms can be assigned to the graphs. The step by step execution of the chosen algorithm can be followed in the displayed pseudo code and the result of each step is directly shown in the graph. A programming interface offers the possibility to implement additional algorithms in Java, Python or the build-in programming language CindyScript. Complete constructions can be exported to a Java applet which can be integrated in arbitrary websites. This allows the development of own interactive teaching units on graphs and graph algorithms for classroom use. Two ready-to-use teaching units on "shortest path" and "Euler tours" fitting to the Berlin curriculum are presented exemplarily.

Name	**Karl Josef Fuchs and Eva Vasarhelyi**
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Title	**Basics in Imperative Programming with CAS**
Working Group	Informatical Concepts and CAS

Abstract

The authors give an insight into a basic course in imperative programming with CAS Hand Held Technology. From the very beginning the concept is based on structured programming startegies. Essential control structures are represented by Nassi – Shneiderman diagrams consequently. Elementary data structures (integers, strings, lists) are discussed in detail. Different modelling steps from posing and analysing the problem beyond steps of refinement to final execution are indicated considerably

Name	**J. L. Galán, G. Aguilera, M. Á. Galán, A. Gálvez, A. J. Jiménez, Y. Padilla, and P. Rodríguez**
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Title	**CAS + Programming = Mathematical creativity**

Abstract

The field of mathematics covers a wide range of knowledge that shares a specific way of representing reality. It arises from the need to solve certain practical problems and is sustained by its ability to deal with, explain, predict and provide models for real situations and lend consistency and rigor to scientific knowledge. The nature of Mathematics ranges from the logical-deductive character of its finished version to the type of reasoning it requires, to the strong internal cohesion within every field and among different fields.
But in teaching we are most interested in the transmission of mathematical knowledge and the proper training of future generations of mathematicians; that is to say, we are interested in encouraging students to participate in the culture of mathematics, something which does not so much involve possessing any final results as it does learning the "mathematical way of doing things". The acquisition of such "mathematical know-how" is a long and arduous process which must begin with prolonged activities that focus on a series of specific elements in order to create intuitions as a prior step to the process of formalization.

The main goal of this paper is to examine the way that CAS (Computer Algebra Systems) is currently being used in Mathematics courses. Special attention is paid to the strong movement taking place among teachers to change the traditional didactic uses given to these tools mainly combining the power of CAS with the flexibility of programming. We also will describe our experience in the last years when using a CAS (DERIVE) throughout the development of specific programs by the students in order to solve typical exercises of the involved subjects.

Name	**Bulent Guven and Adnan Baki**
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Title	**Integrating spherical geometry into geometry curriculum by means of Spherical Easel program**

Abstract

The geometry for prospective mathematics teachers in Turkey is primarily based on Euclidean concepts and constructions. So, prospective mathematics teachers have graduated without making acquainted with different geometries. Prospective mathematics teachers who cannot get acquainted with different geometries perceive Euclidian Geometry as unique geometry that explains all the phenomena and they cannot comprehend the axiomatic nature of geometry. In this way, they cannot develop a multiple thought approaches towards mathematical knowledge. Before all, they cannot have sufficient knowledge about the world we live on (because, the world we live on is approximately a sphere). Because of this, we have integrated spherical geometry into geometry lessons that we have thought in Karadeniz Technical University since 2005. Although we introduced spherical geometry to students at first year, we could not design an effective learning environment for the students to explore spherical geometry because the lack of visual materials to teach spherical geometry. Due to the students were passive recipient of knowledge throughout lessons, Expected attention and success weren't observed. In fact, Spherical geometry is an important area for students to explore new relationships and we knew from our former Euclidian Geometry lessons that prospective mathematics teachers enjoyed exploration based lessons using dynamic geometry software. So, at the beginning of this semester we decided to use dynamic geometry software in spherical geometry lessons as in Euclidian geometry lessons. It was decided to use "Spherical Easel" program developed by David Austin and Will Dickinson in lessons because of its powerful features to study spherical geometry. It was observed that the students joined in lessons with a great pleasure. Furthermore, informal observations showed that the students could explore the mathematical relations easily and the program was an important tool to motivate the students to prove these explorations. In this study, it will be explained how we used "Spherical Easel" program in the spherical geometry lessons.

Name	**Salah Haggag**
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Title	**Computer Algebra Systems in General Relativity**
Working Group	General Session - Other Topics

Abstract

General Relativity (GR) is Einstein's theory of gravitation in which spacetime is a 4-dimensional Riemannian manifold. The space-time geometry is determined by the distribution of the gravitational energy, and the motion of celestial bodies is determined by the spacetime geometry. The interaction between the spacetime and the gravitational energy is described by field equations and equations of motion in tensors form. Hence, the study of GR involves a large number of problems requiring very tedious, time-consuming, and error-prone algebraic manipulation. For this reason, GR was one of the earliest fields of application of Computer Algebra Systems (CAS). Besides using general-purpose systems, many specialized systems and packages have been developed. This paper presents a review of the use of CAS in GR research and teaching. On one

hand, the impact of using CAS in GR research is illustrated by pointing out some important achievements in the field. In particular, by using CAS, the author has been able to obtain results that would have been almost impossible otherwise. On the other hand, CAS can be a very helpful tool in teaching and learning GR. Some projects for using CAS in teaching GR are described.

Name	**Roman Hasek and Vladimira Petraskova**
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Title	**Teaching of financial mathematics using Maple**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

The paper deals with the application of computer algebra system Maple in teaching of financial mathematics. In the Czech Republic financial mathematics is included in curricula of grammar and secondary schools. Therefore this subject is taught at pedagogical faculties too. Most of concepts of financial mathematics are difficult to understand for students. In the paper we show ways of facilitation understanding these concepts using tools of Maple. The main result is in preparing special maplets which enable interactive studying of principles of such concepts. Each of these maplets is devoted to particular financial problem from real life. For example mortgage credit, consumer credit, credit card etc.

Name	**Djordje Herceg and Dragoslav Herceg**
Affiliation	Faculty of Science and Mathematics, Novi Sad, Serbia
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Title	**Numerics with GeoGebra in High School**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

We have prepared a suite of motivational examples to illustrate interpolation, numerical integration, equation solving and initial value problems. Fixed point iteration, Newton's method, primitive quadrature rules, Newton-Cotes formulas, Euler's method and improved Euler's method are implemented as interactive GeoGebra drawings. Our two years' experience in teaching in "Jovan Jovanoviæ Zmaj" high school in Novi Sad will be presented.

Name	**Gábor Horváth, Antal Joós, and Bálint Nagy**
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Title	**WMI is in use**
Working Group	General Session - Other Topics

Abstract

The WMI (WebMathematics Interactive) is developed by the teachers of the Bolyai Institute at the University of Szeged. In 2006 we investigated how the students of the College of Dunaújváros can use the WMI as a helping tool in the learning of mathematics. We continued this work, namely we asked about 500 students learning mathematical analysis and probability theory in order to familiarize a wide range of students with WMI.

Name	**Anesa Hosein, James Aczel, Doug Clow, and John T.E. Richardson**
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Title	**Black-box, Glass-box and Open-box: Which One for Learning?**

Abstract

It is often suggested that CAS makes students susceptible to the black-box phenomenon (e.g. Hornaes & Royrvik, 2000) in which students unreflectively use the software to get answers, without an understanding of the underlying processes. Some researchers (e.g. Buchberger, 1990) have suggested that a "glass-box" (or "white-box") approach, which allows students to see the intermediate calculation steps, could improve the learning value of the tasks undertaken. Alternatively, an "open-box" approach could be used, in which students have some degree of freedom to tweak individual steps. However, there is limited empirical evidence on the advantages of glass-box or open-box approaches over a black-box approach. For example, the CASIO FX 2.0 study compared a glass-box approach with a black-box approach, but looked only at the effects on procedural knowledge (Horton, Storm, & Leonard, 2004). This paper presents data from a remote observation pilot study which investigated and compared the learning benefits of using black-box; glass-box and open-box software. Students were required to do various expected-value problems based on the Galbraith & Haines (2000) framework of mechanical, interpretive and constructive problems. The initial results suggest that black-box software were good for exploration and investigating issues in all three types of problems. There appeared to be no difference in the use of the black-box and glass-box software. Open-box software whilst initially good in promoting understanding of doing the mechanical problems, was soon found to be tedious. Further explorations of these three types of software are presently being conducted in a linear programming study. Buchberger, B. (1990). Should students learn integration rules? ACM SIGSAM Bulletin, 24(1), 10-17. Galbraith, P., & Haines, C. (2000). Conceptual mis(understandings) of beginning undergraduates. International Journal of Mathematical Education in Science and Technology, 31(5), 651-678. Hornaes, H. P., & Royrvik, O. (2000). Aptitude, gender and computer algebra systems. Journal of Engineering Education, 89(3), 323-330. Horton, R. M., Storm, J., & Leonard, W. H. (2004). The graphing calculator as an aid to teaching algebra. Contemporary Issues in Technology and Teacher Education, 4(2), 152-152.

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Title	**Making a difference in mathematics teaching and learning**

Abstract

Knowledge society can only built by people capable of applying their knowledge in industry, economy and services. This means that they must be capable of understanding relationship between their own profession and other fields of knowledge. Often, the role of Mathematics is neglected because the professionals are not familiar with its relation to their expert knowledge and are even scared of it. One of the roles of educators is to remove their fear. To fulfil this obligation is not possible via teaching Mathematics in its traditional manner.

Using information technology, in particular computers and advanced calculators with built-in Computer Algebra Systems and/or Dynamic Geometry is one of the streams attempting to make Mathematics closer to real life. In some countries, their introduction arrives quickly and is strongly supported by their governments; in others it is quite slow. Slovakia and several other countries in Central Europe belong to the latter group. This is one of reasons why CASIO Company has decided to sponsor a pilot project oriented to intensifying the use of advanced calculators in high-schools of three countries of the region: Czech Republic, Hungary and Slovakia. The schools will be equipped by ClassPads, but the project's aim is not teaching how to handle them and to perform calculations faster. Our aim is not to teach Mathematics in its traditional isolation, rather to highlight it as an integral part of our surrounding world. It concentrates on designing and developing modified teaching approaches based on formulating, analyzing and solving real-life problems from science, economy and business having mathematical background. Instead of memorizing formulas, the student will have an opportunity to solve those using graphing calculators. In this way, Mathematics and the calculators will rather serve as carriers of problem-solving.

During the lecture, first results of the project will be shown.

Name	**Marina Issakova**
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Title	**How does an interactive learning environment affect the students' learning?**
Working Group	General Session - Other Topics

Abstract

T-algebra is an interactive learning environment for step-by-step solving of algebra problems in four areas of mathematics, including linear equations. To make the program more intelligent, our own rule dialogue was designed. Each solution step in T-algebra consists of three stages: selection of the transformation rule, marking the parts of expression, entering the result of the operation. In case of an error, T-algebra immediately informs about the mistake and the student has to correct the error in order to proceed to the next stage. Before distribution of T-algebra to all Estonian schools, we organized an experiment to clarify how the program affects the learning results. The study was carried out in the winter 2007. 7 classes (126 students) of 7th grade (13 years old) from four different Estonian schools participated in the experiment. The classes were divided into experimental classes and control classes. The topic of linear equations was studied in the schools at the time and the experiment began when the topic was completed. The experiment consisted of four 45-minute sessions. In the first session, the students solved a pre-test. In the next two sessions, the students practiced solving the problems of the same topic (linear equations). The experimental groups practiced solving these problems with T-algebra, while the control groups practiced solving exactly the same problems using traditional instruction technology – paper and pencil. In the last session, the students solved a post-test. This article thoroughly describes conditions of the experiment and represents the results of pre-test and post-test of experimental and control groups, compares them, presents interesting findings of this comparison and answers the question: How does an interactive learning environment affect the students' learning? The article also examines the differences of the results between schools/teachers and the types of errors in post-test in experimental and control groups.

Name	**Daniel H. Jarvis**
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Title	**Dynamic Geometry Endorsed through Provincial Curriculum, Support Documents, and Professional Development Initiatives: A Digital Perspective from Ontario, Canada**
Working Group	General Session - Other Topics

Abstract

The dynamic Geometer's Sketchpad (GSP) software created by Nicholas Jackiw has been informally endorsed by the Ontario Ministry of Education (OME) in Canada by virtue of its inclusion, by name, within the newly-revised K-12 Ontario Mathematics Curriculum (2005). Further, several key mathematics educational support resources that have been created and made available by OME to Ontario teachers via the internet and through professional development events offer pre-constructed GSP sketches designed to be used by Grades 7-10 students for mathematical explorations. This laptop/poster presentation will direct the viewer to these resources and allow for experimentation with the pre-constructed GSP sketch samples.

Name	**Tolga Kabaca and Muharrem Aktumen**
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Title	**Visulaization of the concept of polar functions by Maple**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

In this study, an interactive worksheet has been developed to make the concept of "graphs of function defined by polar coordinates" more understandable. There are some similarities and differences between Cartesian and polar coordinates of a point in the IR2 plane. Both coordinates are used to define the address of a point in the plane. Only difference is the defining style. It is very hard to explain visually on the blackboard. It can be also easily observed that it is very hard to comprehend for students. We have tried to design an interactive worksheet in Maple that provides; • Animated graphs that shows how a polar defined function is being formed and how a Cartesian defined function is being formed. • Visualization of the polar corresponds of a Cartesian defined function. • Visualization of the Cartesian corresponds of a polar defined function. • A table that shows the polar and Cartesian points of specified function. • Opportunities for students to make all kind applications of polar defined functions.

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Title	**Application of Computer Algebra Systems in Automatic Assessment of Math Skills**

Abstract

Mathematics is one of those areas of education, where the student's progress is measured almost solely by testing his or her ability of problem solving. It has been two years now that the authors develop and use Web-based math courses where the assessment of student's progress is fully automatic. More than 150 types of problems in linear algebra and calculus have been implemented in the form of Java-driven tests. Those tests that involve symbolic computations are linked with Mathematica computational kernel through the Jlink mechanism. An individual test features random generation of an unlimited number of problems of a given type with difficulty level being controlled at design time. Each test incorporates the evaluation of the student's solution. Various methods of grading can be set at design time, depending on the particular purpose that a test is used for (self-assessment or administrative exam). Each test is equipped with the correct solution presentation on demand. In those problems that involve a considerable amount of computational effort (e.g. Gauss elimination), additional special tools are offered in a test window so that the student can concentrate on the method of solution rather than on arithmetic computations. (Another obvious benefit is that the student is thus protected from the risk of frustrating computational errors). Individual tests can be combined into comprehensive exams whose parameters can be set up at design time (e.g., number of problems, difficulty level, grading system, time allowed for solution). The results of an exam can be automatically stored in a database with all authentication and security requirements satisfied.

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Title	**CAS-aided Visualization in LATEX documents for Mathematical Education — Achievements of KETpic**
Working Group	General Session

Abstract

We have been developing *KETpic* as a macro package of a CAS for drawing fine LATEX-pictures, and we use it efficiently in mathematical education. Printed materials for mathematics classes are prepared under several constraints, such as "without animation", "mass printings", "monochrome", and "without halftone shadings". Because of these constraints, visualization in mathematical education tends to be unsatisfactory. Taking full advantages of LATEX and CAS, KETpic enables us to provide teaching materials with figures which are effective for mathematical education. The effects are summarized as follows:

1. The plottings of KETpic are accurate due to CAS, and enable students to deduce mathematical laws.
2. KETpic can provide adequate pictures for students' various interest. For example, when some students who understand a matter try to modify it, then KETpic can give them appropriate and experimental figures.
3. Even though CAS can draw 3D-figures beautifully and automatically, it is expensive for mass printings and the figures are sometimes not easy to understand. Oppositely, 3D-graphics by KETpic are monochrome, but are richly expressive.

In this paper, we give various examples of LATEX-pictures which we drew by using KETpic. For instance, the picture which is used in order to explain the convergence theorem of Fourier series makes it easier for students to understand the idea that function series converge to another function. Also the picture of skeleton is endowed with clear perspective. KETpic gives us great potential for the teaching of combinatorial mathematics. Through these examples, we claim that KETpic has great possibilities of rich mathematical expressions under the constraints above mentioned.

Name	**Piroska B.Kis**
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Title	**Computer aided education of numerical methods**
Working Group	General Session - Other Topics

Abstract

The area of the numerical methods includes elaboration of algorithms, mathematical and computer methods to follow through the theoretical results of the mathematics into the practice. Nowadays, computers are used for realization of the algorithms and methods in most of cases. The development at the informatics and software tools also causes the changing of the teaching methods. While a decade ago, students developed the algorithms using a programming language, but nowadays in addition to they have to use also the facilities of the computer program packages. At the same time, the contents of the electronic libraries keep growing. So, a huge knowledge may be reached on the Internet. The contribution would present the experiences about teaching of numerical methods for BSc students. The teaching is based on lectures and PC labour works. The students learn not only the theory of a topic, but they do how to realize the algorithms, as well as how to use the MATLAB program package for solving their problems. Furthermore, services of electronic libraries and other web sites may be used as a teaching aid. The presentation would also outline the students' attitudes toward the computer aided education.

Name	**Mihaly Klincsik and Csaba Sárvári**
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Title	**Comparisons of capabilities of the CAS and spreadsheets via the statistical quality control**
Working Group	General session

Abstract

Theory of statistical quality control is a good choice to compare the capabilities of the computer algebra systems and the spreadsheets. The actuality of this field is given by the "Processcontrol" package is now available from the Maple 10. We give some practical advices how we can teach the calculations and the visualizations of the control charts and operating curves of the acceptance sampling using the Excel spreadsheet and the Maple 10 computer algebra system.

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Title	**A controlled polygon plotter procedure**

Abstract

The main computer algebra systems (MAPLE, Mathematica, etc.) devote greater and greater care to plotting figures in two and three dimensions. At the same time CAS have a serious shortcoming in displaying of polygons.
Namely, they settle for the algorithm that decomposes the polygons (given by the list of vertices) into triangles obtained by the diagonals started out from the first vertex and displays these triangles without any further examinations.
This procedure works well only on convex polygons. If the polygon is given by solid coordinates, then the CAS don't check that all the vertices of the given polygon determine a plane.
In order to eliminate this shortcoming we have made two procedures: **cpolygonplot** and **cpolygonplot3d** (realized by MAPLE). We call these procedures controlled polygon plotter procedures. These procedures need the same data structures as the corresponding procedures of the Maple.
The cpolygonplot3d procedure executes the following operations with its input.

1) Check that the first three points determine a plane.
2) If so, then all subsequent vertices must belong to this plane.
3) Check that the ordered sequence of vertices form a simple polygon.
4) If so, the polygon is drawn according the given order of vertices.
5) If the ordered sequence of vertices doesn't form a simple polygon, then just the contour of the figure will be drawn and the place and the characteristics of the mistake will be indicated.

The cpolygonplot procedure works in the same way as the cpolygonplot3d procedure apart form the first checking.
The key of these procedures is that they decompose the given polygon into triangles on a right way and these triangles will be passed on the corresponding Maple commands: polygonplot and polygonplot3d
These procedures are used by Euler3D, which is Hungarian development free software.

Name	**Temel Kosa, Adnan Baki, and Bulent Guven**
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Title	**Using dynamic geometry software Cabri-3D to improve students' spatial skills**

Abstract

Spatial ability may be defined as the ability to alter, rotate, bend, generate and transform well-structured visual images. Nowadays spatial abilities are more and more important because we are faced with various spatial problems in technological world. In this study dynamic geometry software Cabri 3D was used to teach Solid Geometry to prospective mathematics teachers. Research carried with two groups. Each of these groups consists of 40 students. In the first group implementations were carried with Cabri 3D during five weeks and the lessons at the second group were thought with traditional ways on blackboard. At the start of the study, Purdue spatial visualization test was used as the pre-test to evaluate the students' spatial visualization skills. After the implementations a 30 items test developed by researchers was applied to two groups as post-test. After the implementations, data were analyzed with ANCOVA. The results of the research showed that dynamic geometry software Cabri 3D is effective tool for developing students' spatial skills because it allows users to rotate the shapes on the screen, to turn the illustrations inside out and etc. In addition, the informal observations of researchers showed that the students in experimental group enjoyed the 3D activities.

Name	**Zoltán Kovács**
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Title	**Blind vs. wise (use of technology)**
Working Group	The impact of CAS-DGS on mathematics teaching

Abstract

During my courses for mathematics major students I often use technology linked to the arising problems. In such cases I have noted that some students were used to learning just some procedures, which make them able to solve (partially) some problems and when they get the result, they accept it passively and do not relate them to the initial problem.
In my note live examples will be performed, originating from my experience about the dangers of blindly using technology and I outline a strategy about how to develop a more critical attitude towards the results from technology.
I believe that wise use of technology offers a modern method in teaching mathematics, without reducing the students' mental contribution.

Name	**Zsolt Lavicza**
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Title	**A comparative study of the use of Computer Algebra Systems in teaching university mathematics in Hungary, United Kingdom, and United States**
Working Group	General Session

Abstract

Recent studies suggest that the increased availability of technology in classrooms has not significantly expanded its use in mathematics teaching and learning. Furthermore, it is suggested that educational technology could diminish the role of teachers. Yet, upon further exploration, it is evident that several competing claims emerge. Firstly, teachers can be the conduit for effective ICT use within classrooms. Secondly, teacher expertise and guidance in the use of technology in educational settings is becoming more significant. Thirdly, teachers' work is becoming more complex and challenging as they are expected to offer well designed technology-enhanced lessons and assessment. Therefore, it is important to examine teachers' views and the decisions they make when choosing to use technology in their teaching. The majority of such research focuses on the use of technology in primary and secondary classrooms while few studies have examined university settings despite the rapidly changing higher education environment. The paucity of such university-level research led me to the development of a two-phase study. The first phase of the study was a qualitative investigation utilizing semi-structured interviews and classroom observations of 22 mathematicians in Hungary, UK, US. Based on the results of this phase a quantitative study was developed in which a questionnaire was sent to 4,000 mathematicians in these three countries. In this study, I focused on Computer Algebra Systems (CAS) as the most important software application in universities. This investigation attempts to uncover why or why not mathematicians choose to integrate CAS into their classroom teaching, what influences their decision for using CAS, how they envision the role of technology of mathematics classes in the future, and how they believe CAS will shape mathematics teaching. My talk will outline the study's results, presents factors that influence CAS integration and highlights differences and similarities among the three participating countries.

Name	**Peter Lebmeir and Jürgen Richter-Gebert**
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Title	**Recognizing the type of a locus with scriptable dynamic geometry Software**
Working Group	Future Trends in Interactive Geometry

Abstract

In the talk we will report on work in progress on algebraic curve recognition. We will exemplify the mathematical background of this topic. We will also demonstrate how concrete implementations can be

generated easily, using a mathematical oriented scripting environment of a suitable DGS. We consider curves that are generated by a mathematical computer program like a computer algebra system or a DGS. The curves are given by a collection of numerical sample points on the curve, usually with high arithmetic precision. The curve generating program itself is treated as a black box that does not provide a priori knowledge on the curve. The ultimate goal of our research is an algorithm that is able to detect the algebraic degree, a parameter set, and if possible a classifying name of the curve (like limacon, lemniscate, Watt curve,...). The talk will describe how this algebraic data can be reconstructed from the sample points, how invariant properties of the curve can be extracted and how randomization techniques can be used to stabilize the results. We demonstrate how this mathematical background can be complemented by a practical implementation. For this we use a DGS with integrated possibilities for mathematical scripting.

Name	**István Lenart**
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Title	**Comparative geometry in E-learning**
Working Group	General Session

Abstract

Description of experiences in teaching comparative geometry for an e-learning course. The participants were future and practising teachers. The present paper gives examples of students' solutions which mainly refer to differences between traditional teaching and e-learning environment. Our experiences in a new area of geometry teaching affirm and reinforce well-known findings and theories in e-teaching and learning.

Key words: comparative geometry on plane and sphere; construction of an e-learning material; manipulative devices in e-learning environment; role of manipulation, illustration and language; transformation of the student-teacher relation and communication.

Name	**Dmitri Lepp**
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Title	**Distribution of Student Mistakes between Three Stages of Solution Steps in Case of Action-Object-Input Solution Scheme**
Working Group	General Session - Other Topics

Abstract

T-algebra is an interactive learning environment for step-by-step solving of algebra problems in four areas of mathematics, including solving monomial and polynomial simplification problems. For better diagnosis of student mistakes, we introduced an action-object-input scheme where the student enters not only the resulting expression after each solution step but also selects a transformation rule and the objects he wants to apply the rule to. When teachers check work on paper, they often search for two non-equivalent expressions in wrong solutions, but it could be very hard to figure out the exact cause of an error. The real cause of the error could lie in a wrong selection of the transformation rule or objects (sub-expressions). In T-algebra we are able to diagnose such errors before the student enters the result of application of the rule, preventing the student from making unnecessary calculations for a clearly wrong result and memorizing the wrong solutions or transformations. Prior to designing the transformation rules for monomial and polynomial simplification problems, we conducted a study among different groups of students. We collected their works on paper and tried to identify different mistakes that the students make in their solutions as well as the causes of mistakes or at what stages of the step these mistakes were made. Later we collected information on student errors in T-algebra from student solutions during the first series of trials. This article describes the results of mentioned studies: what mistakes are made by the students, what errors are made at each stage of solution step, how many errors can be diagnosed before the input stage, etc. The article also compares distributions of errors between stages of the solution step on paper and in T-algebra.

Name	**Aleksandar Lipkovski**
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Title	**Some examples of educational use of CA and DG software**
Working Group	General Session - Other Topics

Abstract

Recently there have been many efforts in the educational use of mathematical software on all education levels: elementary, secondary and higher. Computers are being applied in various essentially different ways: mostly as a calculation helper and as a visualisation device. Though the usefulness of the first approach in the cognitive process, although very important in some instances, is disputable, the second one has proven to be a useful additional tool. There are many commercial software packages developed for the purpose, from small portable free- and shareware to huge and expensive packages such as Wolfram's Mathematica. The author has been using the visual capabilities of the latter in the visualisation of certain algebro-geometrical concepts, concerning plane complex algebraic curves and topology (see www.mi.sanu.ac.yu/vismath/lip and www.mi.sanu.ac.yu/vismath/lipk). In this talk some of author's efforts will be presented, some graphing techniques will be explained, and author's point of view on applicability of CA and DG in education will be discussed.

Name	**Matija Lokar**
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Title	**CAS, CDGS, Wikis... and teaching resources**
Working Group	Informatical Concepts and CAS

Abstract

Some of the crucial milestones of successful incorporation of information and communication technology into math teaching are various aspects of teaching and learning materials. A survey, conducted within teachers' groups, has shown that teachers do not use enough of the resources made available. Especially math teachers were "slow to adopt" such materials. Nevertheless they claim they need suitable teaching resources in order to be able to incorporate information technology into their teaching process. The quality of electronic teaching materials (ETM), the problems with distribution of the materials, too demanding ways of modifying them, and lack of proper classification are only some of the most often mentioned as being the most important reasons for giving up or stopping the usage. The possibility of modifying ETM is one of the properties ETM most often lack and math teachers demand. An important fact is that materials must have the possibility to be changed and adapted to the teacher's own teaching style. Math teachers, especially those teaching in upper primary and secondary schools, do not like using close form solutions; they want to be in control of the whole process. Simple and not too comprehensive ETMs are found most valuable. As already shown elsewhere, using various worksheets based on Computer Algebra Systems (CAS) or Computer Dynamic Geometry Systems (CDGS) can serve as a ubiquitous tool in everyday math teacher's work. We should focus more on producing, collecting and spreading such "small", "simple" worksheets, where CAS and CDGS silently serve as tools to perform observations, to speed up lengthy calculations… On the other hand, Wikis offer a very convenient way for distribution and collection of such materials. By its design alone, a Wiki encourages participation of all users. In this sense, it invites the teachers to become truly involved in the materials they use, as they can modify and improve the course materials on their own. In the talk we will discuss ideas for establishing such Wikis and their influence on the usage of CAS and CDGS in the teaching process. We will show some preliminary examples from a Wiki based site with various CAS (as well as with some CDGS) related resources targeting mostly teachers in upper primary and secondary schools.

Name	**Matija Lokar, Vladimir Batagelj, and Iztok Kavkler**
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Title	**Calibrate and CAS/CDGS materials**
Working Group	General Session – Other Topics

Abstract

The CALIBRATE (Calibrating eLearning in Schools) project brings together eight European countries to carry out a multi-level project designed to support the collaborative use and exchange of learning resources in schools. Its main aim is to provide brokerage system among national repositories of educational materials. As a part of this project we connected Slovenian Educational Network (SEN) to the brokerage system. Quite substantial amount of electronic teaching resources obtainable through SEN are various worksheets based on Computer Algebra Systems (CAS) or Computer Dynamic Geometry Systems (CDGS). But research of SEN usage has shown that math teachers were "slow to adopt" such materials. The quality of electronic teaching materials (ETM), the problems with distribution of the materials, too demanding ways of modifying them, and lack of proper classification are some of the most often mentioned as being reasons for giving up or stopping the usage. In the talk we would address some of these issues, connected to CAS/CDGS materials. One of the quite important issues is the guidelines about the metadata CAS/CDGS resources in the repositories should be equipped with. We will report on two major guidelines the resources in Calibrate should follow. First, they shouldn't contain too much text as such resources don't "travel well" across the borders and tend to be boring to the pupils. Second, what teachers need most, are the tools that assist them in motivating the class. In general, the resource type that best fits both requirements is animations, especially interactive animations. In maths education, CAS/CDGS can be used to produce this kind of resources without forcing the author to master difficult tools outside of her field. We would report on some of the first results of the Calibrate projects used on SEN with connection to CAS/CDGS ETMs.

Name	**Kate Mackrell**
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Title	**Cabri 3D: an environment for creative mathematical design**

Abstract

Cabri 3D is a relatively new software which has great potential in the teaching and learning of both 2D and 3D geometry, in enhancing student ability to visualize, and in modeling and exploring both static and dynamic structures. This paper illustrates these possibilities briefly and then reports on ongoing design research with grade 7 and 8 students in an Ontario school where tasks, materials and approaches are being created to enable students to essentially treat Cabri 3D as a microworld, using mathematically mediating tools to pursue their own goals. It has been found that students are strongly motivated to work with the software when engaged with tasks such as creating a model of their ideal home and are able to design and create a variety of interesting structures. With regard to the instrumental genesis of the software, short Flash demos illustrating how to construct specific objects such as a door that opens have been found to be more useful to students than either worksheets or teacher instruction. However, questions arise concerning the extent to which students will be able to design novel structures and also concerning the way in which mathematical meaning is mediated by such activities.

Name	**Svetlana Maletin and Verica Govedarica**
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Title	**Teaching the Analitic Geometry by means of GeoGebra**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

The present paper describes GeoGebra as a simple and interesting tool suitable for teaching Analytic Geometry, especially in the case when a student first encounters conics. We are going to discuss some methodical reasons for the introduction of GeoGebra into teaching, with the purpose of solving problems. On the other hand, we will show its great possibilities in the case of selecting exercises.

Name	**Frantisek Mosna**
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Title	**Websites for the teaching of mathematics with dynamics figures**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

Electronic forms of teaching are very common and favourite at many schools and the Czech University of Life Sciences in Prague is not an exception. Recently I put together a teaching material intended especially for students of tropical and subtropical agriculture at our university. The aim of this instrument is to improve comprehension of certain concepts, topics and methods of mathematical analysis and linear algebra taught in a basic course of mathematics. The websites contain pieces of basic theoretical knowledge, many step by step opening examples and some exercises for individual work. Besides html, javascript, applets and other usual tools, the typesetting system Latex (and convertor latex2html) was utilized for this purpose. Also, geometrical programme Cabri (and cabriweb) is worth of special attention. This tool enables to create dynamic interactive geometrical figures. User can move some objects (points, lines etc.) and all construction is changing together with them. Such figures are very useful for demonstration of some concepts of differential calculus. For instance, limits can be illustrated by graphs of some functions where student can move the endpoint of an arrow on the x-coordinate and the corresponding arrow on the y-coordinate directs to the limit value. Also, the derivative can be introduced as the slope of tangent to the graph of a function in a given point by means of a figure where secant approaches to tangent. Statistical investigation (McNemar test) and acceptance of students reveal positive influence of this figures (and whole websites as well) to the teaching process and comprehension of students.

Name	**Vladimir Nodelman**
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Title	**Common Software for Mathematics Education**

Abstract

The mathematics internal integrity emphasizing always was one of the important educational efforts. From the other side, the software used in mathematics teaching is separated by application areas: Geometry is studied with Dynamic Geometry Systems (DGS), Algebra and Calculus with Computer Algebra Systems (CAS), Statistics via Dynamic Statistics Systems (DSS) etc. The theme of current conference- "Computer Algebra- and Dynamic Geometry Systems in Mathematics Education"- explicitly confirms this fact. Such multitude of educational aids disturbs teaching because of disintegration both studied subjects (Mathematics Contents) and software interfaces (GUI). This paper illustrates a unified software support by means of "VisuMatica"- a tool developed for teaching mathematics in integrated manner.

Name	**Reinhard Oldenburg**
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Title	**Learning with a linked Computer Algebra and Geometry System â€" The case of curves**
Working Group	Future Trends in Interactive Geometry

Abstract

FeliX is a system that integrates computer algebra and dynamic geometry into a single system that offers multiple linked representations. This opens up new modes of interaction with the mathematical concepts realized in the computer. At a basic level this interaction concerns simple geometric objects like points, segments, lines or circles and their coordinates and equations relating them. Similar to the level leap between analysis and functional analysis there is the interaction with objects of higher complexity: curves. They present some fundamental technical and mathematical challenges for the construction of multi-representational system and, even more interesting, open up even more fields for explorative learning. The presentation will explain the technical issues raised by the treatment of curves and the way they are resolved in the author's system FeliX. Different conceptions about curves are discussed and it is shown how they can be developed using activities with this new kind of software.

Name	**Ildiko Perjesi and Csaba Sarvari**
Affiliation	University of Pecs, Hungary
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Title	**Teaching Fourier series, partial differential equations and their applications – in less time with more CAS**

Abstract

The University of Pécs, Engineering Faculty has launched a two-level educational system, in conjunction with the Bologna process, in 2006. This system allows students having fewer mathematics contact lessons in the basic level in spite of the required mathematics themes were not reduced. The curriculum obliges the discussion of the concept of the Fourier-series and partial differential equation at the end of students' third semester. All lessons are taught in computer laboratories in this semester so that students get acquainted with the basic of use of CAS. In our talk we will outline a particular way of teaching Fourier-series and partial differential equation with assistance of MAPLE. We emphasize the importance of visualization and applicability of the solutions over proofs of theorems. We hope that in this way students will acquire such knowledge that enables them to solve engineering problems with help of CAS in their future studies or workplace. Even thought they don't learn the proof of theorems they are hopefully become more capable problem solvers through their engineering career and can connect mathematics with practice.

Name	**Mladinic Petar, Jelena Gusic, and Zeljka Milin Sipus**
Affiliation	V. gimnazija, Zagreb, Croatia
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Title	**How Does the Geometer's Sketchpad Uncover Complex Numbers in Space**
Working Group	General Session - Other Topics

Abstract

How Does the Geometer's Sketchpad Uncover Complex Numbers in Space Or the "Euler helix" The proposed paper offers alternative approach to visualisation of complex numbers. We connect trigonometric notation of complex numbers with methods of descriptive geometry - we apply Monge's idea of representing space objects by their plane projections. Starting from the question of visualisation of a complex number that is given in the trigonometric (Euler) notation, we represent first its real and imaginary part as the functions of an angle. We represent them in the horizontal (ground) plane (the real part of a complex number) and in the vertical (backdrop) plane (the imaginary part of a complex number). In the side view, a circle representing complex numbers of a given modulus is obtained. Methods of descriptive geometry enable us to determine points in space described by their projections, and the Geometer's Sketchpad's feature "locus" enables us to see them. The obtained space curve is the ordinary helix, for this purpose named the "Euler helix". 3D interpretation embraces all three aspects of a complex number: modulus, the real part and the

imaginary part. It enables us to visualize easily operations in trigonometric notation of complex numbers: multiplication, division and rising to a power. Although this investigation was focused on exploring visualisations of complex numbers, another concepts in secondary school mathematics were investigated - visualisation of wrapping a line around the circle and the concept of a parameterization of a space curve (helix). The results are obtained by vast use of software. It enabled us to uncover the helix and what stands behind it. In visualisation, the possibility of software to colour parametrically was of special importance. It enabled us to visualise complex numbers with certain properties (for e.g. positive real part). The paper is accompanied by the Geometer's Sketchpad's file in which all features can be seen dynamically.

Name	**Valentyna Pikalova and Manfred J. Bauch**
Affiliation	Kharkiv National Pedagogical University, Ukraine
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Title	**Geometric Construction in Real-Life Problem Solving**
Working Group	General Session - Other Topics

Abstract

Geometric construction by ruler and compasses has been a fascinating mathematical problem since ancient times. Today it is still one of the fundamental topics in teaching and learning geometry, and also mathematics. Visualization, spatial reasoning, and geometric modelling are essential skills in problem solving. Implementation of open real-life problems in school environment helps students actively build new knowledge from their own experience and prior knowledge; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life. As an example of modelling such a real-life situation we suggest considering dynamic learning environment DLE "Design the garden". The main aspects are: - interdisciplinary connection – mathematics, art, history etc. - historical background, different cultural traditions - modelling a real-life situation based on experience in geometrical constructions We discuss the development of the DLE in particular within the aspect of the use of different dynamic mathematics software (in our case: DG and GEONExT). The purpose of the DLE is to enhance both German and Ukrainian curricula. The pedagogical basis is given by the synergy of two pedagogical models: a two-step problem-solving strategy and the pedagogical concept I - You – We.

Name	**Rein Prank**
Affiliation	University of Tartu, Estonia
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Title	**Automatic assessment possibilities in problem solving environment with detailed diagnosis of errors**
Working Group	General Session - Other Topics

Abstract

The paper describes how the automatic solution procedure and detailed error diagnosis of problem solving environment T-algebra can be used for automatic assessment of student solutions. T-algebra is an interactive learning environment for exercises in four areas of basic school mathematics: 1) calculation of the values of numerical expressions; 2) operations with fractions; 3) solving of linear equations, inequalities and linear equation systems; 4) operations with monomials and polynomials. The student solves the exercises step by step. Each step consists of three sub-steps: 1) selection of the operation (conversion rule) from the menu, 2) marking the operand(s) in expression, 3) entering the result of operation. The program contains an automatic solver that implements for each problem type the same algorithm that the students are expected to use. We have two possible ways to count the positive component of the mark (how far the student has progressed in creating the solution). For the first (static) way, the teacher has to assign weights to each solution step in the "official" solution algorithm for particular problem type. The program is able to determine what steps were done and what steps were not. For the second (dynamic) way, the program can count the number of applications of T-algebra rules in the full solution created by T-algebra and the number

of steps between the final stage of student solution and the required answer. The negative component can be found by assigning penalties for each error type and for asking hints from the program. T-algebra classifies student errors at three sub-steps of solution steps in 20 error types: application of selected rule is impossible, …, marked sub-expression is not suitable for selected rule, …, wrong sign, numerical calculation error, …, misunderstanding of program requirements.

Name	**Judith Preiner and Markus Hohenwarter**
Affiliation	Florida Atlantic University, USA
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Title	**Creating and Implementing Multimedia Lessons for the Middle Grades**

Abstract

A NSF Math Science Partnership between Florida Atlantic University and the Broward County School District provides south Florida teachers an opportunity to earn a Master of Science in Teaching degree in middle school mathematics. They attend specially designed evening classes and summer institutes to increase their mathematical and pedagogical content knowledge. One focal point of these classes is the effective use of technology for learning and teaching mathematics. In this talk we present how participating middle school teachers were guided through the process of creating multimedia lessons using a variety of software tools. We give examples of their work and discuss their experiences while using multimedia lessons in their classrooms.

Name	**Igor E. Poloskov and Olga V.Sandakova**
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Title	**Using CAS-Mathematica for teaching mechanics**

Abstract

This paper describes some aspects of educational process for the subject "Computer algebra" in Perm State University (PSU) for students which study mechanics on the base of two-year common mathematical courses The whole process lasts three semesters. The first step is a study (in a lecture room and a computer class) of basic features of Mathematica and principles of this computer algebra system (CAS) exploitation in interactive and program regimes. On this stage, students must solve two individual tasks, which concern with realization of different mathematical algorithms (from analytical geometry, mathematical analysis, theory of ordinary and partial differential equations, numerical analysis i.e.) with the help of Mathematica, and pass a test.

The second stage is a training of students in theoretical mechanics. A number of tasks from this subject are offered to every student and these tasks must be fulfilled on the base of Mathematica knowledge.

The third part named "CAS in mechanics" proposes a deepening of students skills for CAS applications in different topics of theoretical and applied mechanics such as nonlinear and random oscillations, methods of small parameters, problems of optimal control, symbolic-numeric calculations, methods of solution for difference-differential and integro-differential equations and others. Moreover, a short review of some CAS (Maple, Maxima, Derive) is presented. On this stage students must solve three problems and sit for an examination.

We suppose that an inclusion of "Computer algebra" topic in the PSU program in a framework of National project "Education" and an increase of hardware and software facilities will help us to archive new higher level of our educational and scientific activities.

Name	**András Ringler**
Affiliation	University of Szeged, Hungary
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Title	**Geometric solution of Elliptic and Hyperbolic "SYMPTOMAS"**

Abstract

The author found a surprising geometric method to solve quadratic equations. With this method, one can find also the second solution of elliptic ($a \cdot x - x^2 = b^2$ and $a \cdot x - x^2 = -b^2$), and hyperbolic ($a \cdot x + x^2 = b^2$ and $a \cdot x + x^2 = -b^2$) "symptomas" (equations).
The geometric solution of these quadratic equations will be shown during the presentation with the help of a computer animation. The most important information (the geometric condition of solubility) are summarized in Tab.1.
The presented animation can be found and seen on the next website:
www.mozaik.info.hu/homepage/mozaportal/matematika.php
To see the four animations use the next steps! To activate the program, first click on "**Gondolkozzunk görögül**", then on "**Megnyitás**", then on "**Ringler Andras G** …", and then on the "**GEOM exe**" icon.

Tab. 1.

Form of the equation	The geometric- and algebraic condition of solubility	Number of solutions and their signs
$a \cdot x - x^2 = b^2$	$a > 2b$; the discriminant, $a^2 - 4b^2$, must be positive	Two positive solutions
$a \cdot x - x^2 = -b^2$	There is no geometric condition; the discriminant, $a^2 + 4b^2$, is positive	Two solutions, a positive and a negative
$a \cdot x + x^2 = b^2$	There is no geometric condition; the discriminant, $a^2 + 4b^2$, is positive	Two solutions, a positive and a negative
$a \cdot x + x^2 = -b^2$	$a > 2b$; the discriminant, $a^2 - 4b^2$, must be positive	Two negative solutions

The author is sure that the described new method will give both students and professors much joy, and he hopes that the strict Greek masters do not „roll over in their graves", being angry with him, because he introduced the "$-b^2$" value into their very nice elliptic and hyperbolic "συμπτωμα"-s. The author is hoping that the old Greek masters will forgive him for taking the courage to continue the thoughts they - ≈2000 years ago - began.

Name	**Cvetka Rojko**
Affiliation	The National Education Institute, Slovenia
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Title	**Technology in Modern Curricula**

Abstract

Implementation of technology into curricula can change the perception of "school mathematics". It depends on the role and significance we give to technology, which consequently influences the approach to teaching and constructing of learning environment.
In Slovenia, in the last few years, we have changed some of our curricula and defined a much stronger role of ICT in mathematics lessons. There are two kinds of issues.
One is that students need the knowledge of using the technology in further work or study. A task of mathematics lessons is to offer needed support in developing of this competence. Especially in solving of mathematical problems in their work or daily life by use of technology.
Another issue is applying of technology in the learning process of development of mathematical knowledge. When we use it, especially if we use it regularly, we must be aware, that final mathematical knowledge of students will be different then those that had been used earlier. Therefore we should know, what knowledge and what competences students need to develop and technology should support it.
In my presentation, I will present the role and significance of technology in the current Slovenian curricula. There will be also some practical examples, showing in which way technology can support the development of different kinds of mathematical knowledge.

Name	**Anna Rybak**
Affiliation	Institute of Computer Science, University of Bialystok, Poland
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Title	**Using ICT in math education: How to investigate its influence on students' cognitive achievements?**

Abstract

Computer and information technology become a part of school reality. Students enjoy lessons in the computer laboratory. They are more motivated for learning when the educational process is more attractive and didactical tools are modern. Results of many studies over the world show that using computers with proper educational software has positive impact on students' emotional achievements, including higher motivation and self-confidence, and better attitude to education as a whole. What about cognitive achievements? Does the use of computers during the lessons lead to better understanding of concepts, increased level of knowledge, and higher level of skills to apply this knowledge in problem solving? Are results of class-tests and exams better in this case?
It is difficult to measure the connection between using ICT and students' learning outcomes, because using ICT is never the only factor of changes in students' knowledge and skills. Such study requires special tools. In this paper I present an idea of assessing the influence of using ICT in math education on students' learning outcomes. The experiment has been carried out in a middle school in Białystok.

Name	**Anna Rybak and Istvan Lenart**
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Title	**Play computer game ... and learn spherical geometry**
Working Group	General Session - Other Topics

Abstract

The idea of learning through playing has been known and used in education for centuries. In many didactical situations, different games were used to increase students' interest, make understanding and learning easier, and improve learning outcomes. A game during the lesson may often prove to be a vehicle for the constructivist approach to teaching and learning. Sometimes the use of games is a good way of introducing the concepts from outside the curriculum, concepts that would be theoretically difficult for young children without proper methodological approach. In the age of technology, computer games are more popular among our students than traditional ones. In this paper we would like to present a way from playing a simple computer game to learning mathematical concepts from outside the curriculum – via spherical geometry. In addition, we present how to join using computer game and manipulative didactical tools in the same educational process based on students' creative activity. The experiment has been conducted in Białystok with three groups of participants: math teachers-to-be, students of 5-6 grades of middle school, and practising teachers of mathematics. Results are described in our paper.

Name	**Hakan Sandir, Ahmet Arýkan, and Mehmet Bulut**
Affiliation	Gazi University, Turkey
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Title	**Pre-service mathematics teachers' views on dynamic mathematics software**
Working Group	Research perspectives of the impact of dynamic mathematics on teaching and learning

Abstract

In this study, pre-service mathematics teachers were applied dynamic mathematics software on teaching and learning. For this, students were grouped as 4 persons. Each group prepared lecture notes for teaching

and learning dynamic mathematics software. Researcher helped them during preparation of notes and learning process. Pre-service mathematics teachers were applied this software on mathematics subjects in classroom environment. Also they developed teaching models for subjects. After that, their views about impact of dynamic mathematics on teaching and learning. According to results, pre-service teachers were investigated dynamic software. Their ideas showed that these software are effective tool for mathematics teaching and learning especially on geometry subjects.

Name	**Marvin Schiller, Christoph Benzmueller and Dominik Dietrich**
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Title	**Proof Step Analysis for Proof Tutoring -- A Learning Approach to Granularity**
Working Group	Automated Reasoning and Math Education

Abstract

We present a proof step diagnosis module based on the mathematical assistant system OMEGA. The task of this module is to evaluate proof steps as typically uttered by students in tutoring sessions on mathematical proofs. In particular, we want to categorize the step size of proof steps performed by the student, in order to recognize if they are appropriate with respect to the student model. In a tutoring context, OMEGA can also help the student by suggesting proof steps; therefore identifying the appropriate step size for a given tutoring session is also useful for adapting the level of detail of proof step presentation. We propose an approach which builds on reconstructions of the proof in question via automated proof search using a cognitively motivated proof calculus. By employing learning techniques and a student model, the analysis of the component can be adjusted to different domains and users. We present a snapshot of our prototype under development, which is based on experiences with an earlier case study.

Name	**Mazen Shahin**
Affiliation	Delaware State University, USA
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Title	**Explorations in Mathematical Models of Biological Systems**

Abstract

In this paper we will share the pedagogy and the methodology of a newly developed course: Elementary Mathematical Modelling of Biological Systems. The difference equations and matrix algebra are the main mathematical tools used in this course. Difference equations represent a very sophisticated and powerful tool to model a wide range of discrete time biological systems, while matrices are an excellent tool in modelling linear systems. The methodology of this laboratory course integrates the use of computers and cooperative learning creating an interactive learning environment. In this approach most of the class/lab time is devoted to discussions on carefully prepared activities, where students work in small groups and reflect on what they have done. We emphasize using biological and ecological problems and computers to promote conceptual understanding and to motivate students. In this approach the students interact with their group mates, computers, and the instructor, becoming active participants rather than passive observers. As an illustration of the pedagogical approach we will explain how the students construct a model of population of a species with age structure and use the computers to determine the population's stable age distribution and the rate of change that corresponds to the stable age distribution vector. This model will be utilized to intuitively introduce the Eigen values and corresponding eigenvectors of a square matrix. As another illustration of our approach, we will discuss a practical model of management of a renewable natural resource, where students consider the population of a species that is harvested. The population follows a logistic growth model. We are interested in optimizing and maintaining the economic yield. Finally we will discuss a model of population movement between a city and its surrounding suburbs as well as a model that utilizes Markov chains in genetics.

Name	**Zeljka Milin Sipus, Jelena Gusic, and Petar Mladinic**
Affiliation	University of Zagreb, Croatia
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Title	**The Geometer's Sketchpad as a Learning and Teaching Tool in Croatia**
Working Group	General Session - Other Topics

Abstract

The proposed poster illustrates the activities and methods applied to introduce DGS in classroom environment. In order to introduce DGS in mathematics education in elementary and secondary schools, two basic steps were taken (from the late nineties): activities with teachers and activities with students. The teachers' trainings were organized in three levels: getting acquainted with software, working with students/classroom materials and finally producing new classroom activities – teachers' presentations as well as students' working materials. DGS was introduced to students in their classrooms, where they explored regular school topics by means of technology. Besides, some students were engaged in extracurricular (elective) activities where they worked in group and/or individual projects. Activities from both steps were presented to math community on teachers' meetings, where advantages of use of new methods are discussed and analysed. The poster is accompanied by computer interactive dynamical presentations of presented materials.

Name	**Hans – Stefan Siller**
Affiliation	University of Salzburg, Austria
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Title	**Modelling – A pivotal Idea for Interdisciplinary Teaching in Mathematics and Computer Science**
Working Group	Informatical Concepts and CAS

Abstract

Interdisciplinary Teaching is of increasing importance in school. Just now the two subjects mathematics and computer science which have been closely connected for numerous years offer a plenitude of possibilities for professional implementation. Basic ideas will be necessary to facilitate the different approaches to each subject. One idea which is mostly approved in mathematics and computer science is the concept of modelling. The variable systems of modelling in practice will be demonstrated.
The process of modelling will be shown exemplarily accenting the inevitable informatical step of graphical representation via diagrams. Appropriate CAS features support the steps of modelling. The results are finally interpreted by the mathematician / computer scientist.
Thus the fundamental idea of modelling supports the students' understanding of mathematical and informatical concepts.

Name	**Mária Slavíčková and Ingrida Kraslanová**
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Title	**How to teach functions by using of Derive**
Working Group	Relating procedural and conceptual knowledge of mathematics through CAL

Abstract

In the talk we intend to inform about results of realized experiment concerning the usage of Derive and also about influence of this software on procedural and conceptual knowledge. We will present some Derive-worksheets intended for teaching mathematics at high school. We will also show some examples of using CAS on mathematics lessons at high school and at the University (we will focus on functions and calculus).

Name	**Radivoje Stojkovic, Djurdjica Takači, and Jasminka Radovanović**
Affiliation	Serbia
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Title	**The analysis of the behaviour of functions using computer**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

We shall present the method for the teaching behaviour of two functions using programme packages Mathematica, Scientific WorkPlace and GeoGebra. This can be considered as the contribution to the visualization of limit process. Furthermore, we shall present the analysis of questionnaire which was done among the students who were presented the method in question.

Name	**Alla Stolyarevska**
Affiliation	Mathematics & Computer Science, Kharkov, Ukraine
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Title	**Media education and mathematics**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

We consider the using the computer technologies in education as a part of media education. The computer technologies are a set of methods directed on delivering, processing and storage of the information by means of computer. The concrete methodical recommendations with using the computer technologies are developed at studying a course of higher mathematics in the International Solomon University (ISU) - the not state higher education institutions of Ukraine. ISU was founded in 1991 with orientation onto the modern world standards of higher education. The Eastern-Ukrainian Branch of ISU, which is situated in Kharkov, provides preparation of the students on specialty "Software of automated systems". The grounds of studying are the plates of classical mathematics. One of the new directions in studying connects with the mathematical packages. We apply the mathematical package Derive for performing the analytical and numerical calculations; for constructing the graphs of functions, for tracing the curves and the imaging the surfaces according to their equations. The process of training provides an active students' mastering the material of the course of higher mathematics. The study of mathematics on the basis of mathematical packages we classify as a deductive method that specifies the general way of the decision of different problems. The methods of training on the basis of mathematical packages are realized through the inclusion of the system of laboratory works. They are directed on formation of the generalized conceptual receptions on the basis of stimulation of cognitive activity.

Name	**Djurdjica Takaɥci**
Affiliation	University of Novi Sad, Serbia
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Title	**The introduction of the limits of functions by using computer**

Abstract

The limits of functions are taught in the last grade of high schools in Serbia. This can be considered (by Tall) as the transition to advanced mathematical thinking and the students has difficulties to accept this notion properly. Therefore, the teachers are supposed to be well prepared for such lessons. In this paper we present the introduction of the limits of functions by using the programme packages GeoGebra and Scientific WorkPlace. We consider the graphs of functions which domains cause the necessity of introducing the limiting process. The points which do not belong to the domain of corresponding functions, but they are its accumulation points are pointed out. The behaving of functions at the mentioned points by using computer

are considered. The computer is used "as a microscope" to show the values of considered functions on the very small intervals surrounding the mention points. This presentation can be done, also, as the motivations for the introduction the limits of functions. We consider all possible problems appearing working with limxa f (x) = L
_ if L and a real numbers;
_ if L is a real number and a = _1;
_ if L = _1; and a is a real number;
_ if L = _1; and a = _1:
We also compare the results of questionnaire given to two groups of students, one of them were taught by computer and the other one without it.

Name	**Jelena Tatar**
Affiliation	Maja Jevđeviæ Milutinoviæ High school, Novi Sad, Serbia
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Title	**The influence of CAS on the development of procedural and structural knowledge**
Working Group	General Session

Abstract

Examining the effect of the computer usage in acquiring the knowledge in mathematics in high school mr Jelena Tatar, Maja Jevđeviæ Milutinoviæ High school „Jovan Jovanoviæ Zmaj" Novi Sad We examine the effect of computer usage in acquiring the knowledge in mathematics in high school. We were examining the level of acquired knowledge in the second year of high school (the graphs of functions). The same unit was presented to one group of students in traditional way, while the other group was taught by computers. We were interested to see whether these two groups would show the differences in the level of the acquired knowledge. The groups had been equalized in general skills and previous knowledge in mathematics. We are going to analyse the achievements in tests from the given unit and differences in types of mistakes in both groups of students.

Name	**Philip Todd**
Affiliation	Saltire Software, Beaverton, Oregon, USA
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Title	**Geometry Expressions: an Interactive Constraint Based Symbolic Geometry System**
Working Group	Future Trends in Interactive Geometry

Abstract

Dynamic geometry systems such as Geometers' SketchPad or Cabri are productive environments for the exploration of geometric relationships. They are, however, strictly numeric, and this limits their applicability where the interplay between geometry and algebra are being studied. We present Geometry Expressions – a dynamic symbolic geometry environment. While retaining the ease of use of a typical dynamic geometry environment, Geometry Expressions diverges by using constraints rather than constructions as the primary geometry specification mechanism (fig 1) and by working symbolically rather than numerically. Constraints, such as distances and angles, are specified symbolically. Symbolic measurements for quantities such as distances, angles, areas, locus equations, are automatically computed by the system. We outline how these features combine to create a rich dynamic environment for exploring the interplay between geometry and algebra, between induction and proof.

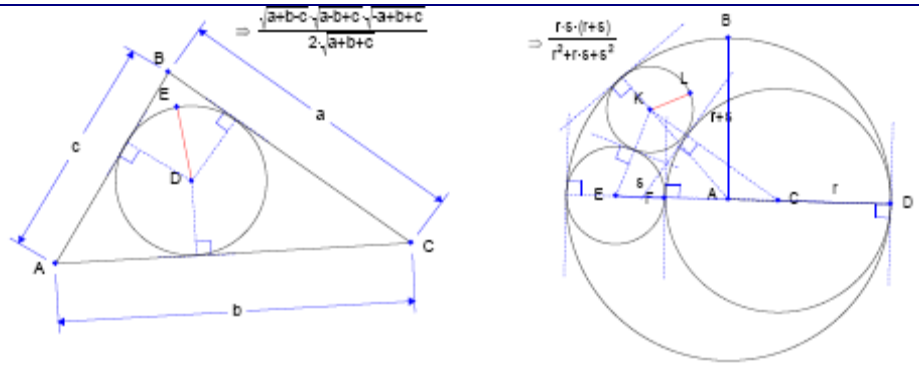

Figure 1: The same tangency constraint defines a circle tangential to three lines and a circle tangential to three circles, although quite different(and quite complicated) constructions would be required to specify these figures.

Name	**Eno Tonisson**
Affiliation	University of Tartu, Estonia
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Title	**Real and Complex Domains in School Mathematics and in Computer Algebra Systems**
Working Group	General Session - Other Topics

Abstract

The available number domain gradually extends for the students during their school time. Students work by default in their largest number domain and they usually do not think about number domain of the values of the variables in expressions, equations, etc. Actually, the topic of number domains is certainly important – there may be different transformation rules allowed or the solution sets may differ in different number domains. Changeover from real numbers to complex numbers is very crucial. The school curricula in many countries normally do not include complex numbers while in other countries complex numbers are a part of the school curricula. Even if complex numbers are introduced, there are only some elementary properties and operations treated. Use of a computer algebra system (CAS) in the learning process creates a necessity and provides a chance to treat real and complex number domains more thoroughly. The imaginary numbers may appear in solutions of equations (already in case of quadratic equation). CAS may provide a solution of equation that is real number but is not appropriate when operating with real numbers only. Equivalences known in school may not hold in CAS because of use of complex numbers. There are examples that teachers (and students who have received little explanation) expect but some examples (logarithms, trigonometry) are less known. The paper scrutinizes the solutions of the school problems where CASs "cross the border" of real number domain. CASs Derive, Maple, Mathematica, MuPAD, TI-92+ and WIRIS are observed. There are differences in the operation of different CASs – in default domain; in determination of domain of the calculation result, the variable value, the equation (inequality) solution or the entire process; in technical approaches (packages, assumptions, etc.). The paper also discusses possible teacher actions in problematic cases – avoid such problem or add explanations (which?) …

Name	**Eno Tonisson and Nadezda Velikanova**
Affiliation	University of Tartu, Estonia
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Title	**Answers to equations from school textbooks offered by computer algebra systems**
Working Group	General Session - Other Topics

Abstract

In the beginning CASs were designed mainly to help professional users of mathematics, and were not directly intended for use in mathematics education, therefore in comparison with schools they may have different notions and standards. Nowadays this situation is improving and more systems are created or modified taking into consideration in particular school problems and needs, but there are still some differences. The paper investigates how well computer algebra systems handle equations from school mathematics. Main aim is to develop the strategy for researching and comparing CASs, and afterwards basing on this strategy to investigate, how well computer algebra systems handle equations from school textbooks. The classification of equations, which was made up for the research, covers majority of the equations studied at school. Totally 60 problems were solved with a help of each of 7 CASs (Derive, Maple, Mathcad, Mathematica, MuPAD, TI-92 Plus and WIRIS). Equations were divided into 8 types (35 subtypes). Done research shows, that mostly systems are reliable and give reasonable answers. Some remarks regarding the obtained answer can be easily explained by built-in standards and notions, which can differ from school assumptions. Investigated 7 systems work quite similarly for tested equations and give practically same results. There are differences in the way of writing down the answers or indicating some situations. Bigger problems were noticed only with three types (irrational equations containing negative number under the root with odd root index, exponential equations solved by taking the logarithm and trigonometric equations).

Name	**Zlatko Udovicic and Vesna Satev**
Affiliation	Faculty of Sciences, Department of Mathematics, Bosnia and Herzegovina
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Title	**Mathematica and Convergence**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

There is no doubt that one of the most important concepts in mathematics is the concept of the convergence. A numerous mathematical objects are defined exactly as the result of the corresponding limiting process (derivative of a function, definite integral, infinite sum ...). It is clear that visualization of the convergence plays very important role in it's proper understanding. We developed an Mathematica package for "numerical and visual" proving of the convergence for different types of sequences. By using this package one can check the convergence of the real sequences (especially the convergence of the infinitive sums), point wise and uniform convergence of the functional sequences and sums, the convergence of the power and Fourier series and the convergence of the vectors in Matlab.

Name	**Robert Vajda**
Affiliation	University of Szeged, Hungary
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Title	**Report on the CreaComp Project: Linking computer algebra and automated reasoning for providing a new learning environment for elementary analysis**
Working Group	Automated Reasoning and Mathematical Education

Abstract

CreaComp is a project at the University of Linz, which aims at producing computer-supported interactive learning units for several mathematical topics at introductory university level. The units are available as Mathematica notebooks. For student's experimentation we provide computational, graphical and reasoning tools as well. This paper focuses on the elementary analysis units. The computational and graphical tools facilitate the exploration of new mathematical objects and their properties (e.g. boundedness, continuity, limits of real valued functions). Using the provided tools students should be able to collect empirical data systematically and come up with conjectures. To formulate a conjecture precisely and to investigate its validity, the Theorema system provides full predicate logic with a user-friendly two-dimensional syntax and a

couple of automated reasoners that produce proofs in an easy-to-read and natural presentation for the user. We demonstrate the learning situations and the provided tools through several examples.

Name	**Emiliya Velikova and Valentina Voinohovska**
Affiliation	University of Rousse, Bulgaria
E-mail	emily@ami.ru.acad.bg
Title	**Computer-aided Scientific Research of Students for Creating Original Mathematical Problems**
Working Group	Computer-aided Experiments and Visualization in Education

Abstract

One of the most important element in training of students (future teachers in Mathematics and Computer Sciences) in the discipline "Method of Transformations" is computer-aided scientific research.
The paper presents different ways of *Maple* and GEONExT usage in the students' experiments for discovering the properties of some geometric transformations and for creating original problems (new geometric inequalities). The comparative analysis shows that the computer-aided students' research provide conditions for a much higher level of development of the knowledge and research skills, than the "traditional" extracurricular activities in the field of mathematics.

Name	**Wolfgang Windsteiger**
Affiliation	Research Institute for Symbolic Computation, JKU Linz, Austria
E-mail	Wolfgang.Windsteiger@risc.uni-linz.ac.at
Title	**Towards Computer-Supported Proving in Maths Education**
Working Group	Automated Reasoning and Mathematical Education

Abstract

In this work we want to present an environment for learning and teaching mathematics that integrates computer-support for different aspects during different stages of the mathematical learning process. In the first phase of confrontation with new content the student can benefit from computer-powered interactive experiments that help to develop and sharpen the intuition about the concepts involved. Experiments are designed carefully such that they lead the user to the discovery of mathematically interesting properties of the investigated objects. During the strengthening phase, the student is asked to first formulate the observed properties in an exact mathematical language and then prove these propositions. At this point, an automated theorem proving system is employed to assist the student in this task.
Interactive elements are based on the symbolic and graphical computation capabilities of the well-known computer algebra system Mathematica, and we use its possibility to connect to Java in order to provide intuitive graphical user interfaces. Algorithms provided by or implemented in Mathematica are employed in these experiments so that a student can not only work with (a few) pre-computed examples but also with user data or randomly generated input. The formal parts are supported by Theorema, a mathematical assistant system also based on Mathematica. Theorema supplies an intuitive two-dimensional input syntax close to common mathematical style and it provides fully-automated or interactive theorem provers for different domains, which generate mathematical proofs in human-readable format.
We illustrate the flavor of human-computer-interaction possible in such an environment in examples taken from the learning units on ``Equivalence relations and set partitions'' and ``Polynomial interpolation'' and claim that the computer in classroom can not only be employed for numerical or symbolic computation or visualization but it can contribute also to the realm of teaching mathematical proofs.

Name	**Otto Wurnig**
Affiliation	University of Graz, Austria
E-mail	adyotto.wurnig@aon.at
Title	**Introduction of the parabola in form 11 with the use of GeoGebra and CAS**

Abstract

The first Research Projects in Austria (1992-2004) organized by the ACDCA (Austrian Centre for Didactics of Computer Algebra) had the goal to reform the teaching of Mathematics with the use of CAS like PC-DERIVE or TI-DERIVE. In Analytic Geometry the two window techniques were used. One window was taken as an algebra window, the other as a geometric window. A new project was started in January 2005. In this project ACDCA, Geogebra and mathe-online cooperate. It is especially Geogebra that offers new possibilities. It is characteristic of GeoGebra that an expression in the algebra window corresponds to an object in the geometry window and vice versa. In my lecture I will demonstrate new possibilities of Geogebra when introducing the parabola in form 11 in two alternative ways:
The traditional way is to begin with the definition of the parabola as a locus of points. The students can now do the construction of a parabola – given focus F and *directrix* l - with GeoGebra. With the command "*Conic through five points*" they can get the parabola line on the geometry window and the equation of the parabola on the algebra window.
But one question still remains to be answered: *What is the relation between the three different conics?*
So I looked for another way which would allow the students to find the parabola exactly between ellipse and hyperbola. I based my work on the knowledge students had acquired in Physics. The students learn that the earth satellites are moving in conics. If the initial speed is exactly 11,2 km/s, the orbit of the satellite will be a parabola. The students are given the picture from the school book in Physics and their task is to draw an ellipse and hyperbola with a=4 and b= $\sqrt{8}$ with the help of GeoGebra in such a way that the vertex lies in the origin point and the x-axe is the common symmetric axe. The parabola as the curve between ellipse and hyperbola can now be found very quickly.
One important goal is to find the general formula of the parabola. To reach this goal the use of a CAS is very useful.

Name	**Nurit Zehavi and Giora Mann**
Affiliation	Weizmann Institute of Science, Israel
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Title	**The tension between parametric registers and explicit patterns**
Working Group	Other

Abstract

The theoretical framework of this study is based on Duval's classification of the various registers of semiotic representations used in mathematical activities (Duval, 2006). According to Duval, the learning of mathematics is supported by processing within the same register (presentation) and by conversion between registers. Here we focus on a special register that we identify as parametric register. A parametric register is implemented in mathematical software in the form of slider bars that enable to demonstrate, in a dynamic way, the effect of a parameter in an algebraic expression on the shape of the related graph. Our findings elicit cognitive activities in the processing of slider bars, and also indicate that the tension created by the conversion between this parametric register and the symbolic (algebraic) register sharpen the way we think about parameters. We chose to explore a problem that was raised in the research and development process of a course in Analytic Geometry using CAS. We implemented slider bars to animate pairs of tangents to a hyperbola. The obtained figures were very informative: The plane is partitioned into four loci: (1) points through which no tangent passes, (2) points through which a single tangent passes, (3) points through which two tangents to the same branch of the hyperbola pass, and (4) points through which two tangents pass, one to each branch. We asked teachers to rate (from 1 to 6) the need to prove algebraically the above results. Afterwards, they were asked to convert the parametric register to a symbolic register in order to make explicit the meaning of the symbolic expressions we obtained by the CAS. These expressions were

obtained while we designed the animation of tangents through a general point P(X, Y). The expressions encapsulate the relationships between the different variables, which need to be unfolded by means of advanced symbol sense. Then we asked the teachers again to rate the need to provide explicit algebraic proof of the partition of the plane into four loci. In the lecture we will discuss how the teachers' views reflect the tension, in their mathematical thinking, between the parametric register and the symbolic register. Interestingly, in spite of our request to refer to the symbolic register for proof, some teachers introduced a geometric register. This conversion should not be surprising, since proof in geometry plays an important role in the mathematics culture of deductive thinking.

Workshops

Title	**An approach to teaching and leaning using program Graph**

Silva Kmetic, The National Education Institute of The Republic of Slovenia

Abstract

Program Graph (http://padowan.dk/graph) is an open coded program. It is PC compatible with Windows operating system 98 up to XP. It is easy to use and very appropriate for beginners in the field of ICT. The workshop task is to present and discuss how teaching and learning of mathematics can be supported with the program. The software usage will be introduced from teacher's and student's perspective. The presentation will include the transition from functional use – what the software can do – to pedagogical use – how the software can be efficiently used in classroom.

1. Teachers can use it for preparing examples, worksheets, and other materials like matching cards, loop cards…
2. The other possibility is that they use pre-prepared files, which are presented at certain moment for different purposes like checking students' work, guiding them through certain procedures, or as a starting point to students' investigations.
3. Teachers can develop simple e- working sheets combining Graph with other software (Excel, PowerPoint, Word), which give feedback information to learners.
4. Teachers develop learning environment in which students develop their own mathematics experiences.
5. Students learn by doing mathematics with wide knowledge of program possibilities.
6. Students learn by using pre-prepared files for investigations, problem solving mathematical or real life problems or for self assessment.
7. Students learn by doing e-working sheets. To summarize, Graph can be problem-solving assistant, visualization and interpretation add and investigative or exploratory add.

The illustrative examples in the workshop support a development of concept of function by three possible representations: by equation – analytical approach, by graph – visual approach or by table – numerical approach.

Title	**Apprenti Geometre**

G. Noel and A. Vandenbruaene, Centre de Recherche sur l'Enseignement des Mathematiques (CREM), Belgium

Abstract

The workshop will present **Apprenti Géomètre**, a dynamic geometry software conceived in order to be used by pupils during at least primary school and junior high school. To be fitted to such a various public, **Apprenti Géomètre** has several levels adaptable to the pupils' knowledge.

As soon as the start of primary school, the soft's level A enables the user to realize jigsaw puzzles with rigid geometric shapes, called "standard shapes". A magnetic property makes it easy to assemble the shapes.

On the screen, the shapes can be moved with the mouse, they can also be turned and returned. They are assembled in families which can be used, for instance, in the context of fraction learning.

They can be cut in pieces which can be glued so that different shapes can be seen to have the same area. This can be extended to deriving the area formulas for special polygons.

There are several sets of standard shapes, and the teacher can build himself new ones, adapted to the subjects he wants to study.

At the soft's level B, i.e. at the end of primary school and at junior high school, the pupil can draw freely and easily general polygons and circles. Predefined shapes exist, such as isosceles triangles or trapezoids, rectangles, parallelograms, as well as segments, lines and strips.

As the preceding ones, these shapes can be duplicated, moved, assembled, cut, glued. Moreover they can be deformed by moving a point, but of course when you move any vertex of an isosceles triangle, the deformed shape remains an isosceles triangle and the figure is automatically rebuilt.

When time comes to learn geometrical transformations, the user finds that the soft enables him to define translations, rotations, symmetries and to build the image of any shape by them. And if he modifies a shape or a transformation, the image of that shape by that transformation is also rebuilt.

There are still other peculiarities to be presented but we limit ourselves to two of them: the soft is free and the user can easily translate it into any language.

Title	**Beyond DGS - Simulations and Scripting with Cinderella**

Ulrich Kortenkamp, University of Education Schwäbisch Gmünd, Germany

Abstract

In this workshop we will explore the new facilities of the Interactive Geometry Software Cinderella.2 and work with physics simulations and custom scripts. A basic knowledge of any geometry software is required. Participants will receive a temporary license for the software and are encouraged to bring their own laptops to work with.

Title	**Connecting geometrical, numerical and algebraic aspects of 3D geometry using Cabri 3D**

Colette Laborde, University Joseph Fourier, Grenoble, France

Abstract

The workshop aims at illustrating how to develop a geometrical sense of equations and numerical formulas for calculating volumes and areas of 3D objects by using the dynamic possibilities of Cabri 3D: varying objects, changing the view angle on objects, geometric constructions and dynamic numerical calculations and equations. Participants will explore themselves in Cabri 3D hands-on activities meant for secondary high school students. Participants do not need a prior knowledge of Cabri 3D. They will be introduced to the friendly interface of Cabri 3D while they explore the progressive sequence of activities.

Title	**Exploring symmetries with secondary school students- an introduction to mathematical concepts**

Jelena Gusic, Milin Sipus, Petar Mladinic, University of Zagreb, Croatia

Abstract

Symmetries of objects form a huge and important subject existing almost everywhere: in nature as well as in arts and sciences. The purpose of this workshop is to show how students can investigate some mathematical concepts underlying the notion of symmetry. Participants in this workshop will be engaged in investigation of symmetries in geometry: starting from the study of geometric transformations, plane isometries and congruences leading to the study of symmetries of plane figures. These geometric notions offer the opportunity of bringing together geometry and algebra; they result in students' deeper insight of some algebraic concepts like functions, their compositions and inverses, and finally a notion of a group. The proposed workshop is based on dynamic geometry software workshops organized for students and teachers of mathematics in secondary schools.

Title	**GeoGebra in Secondary School Teaching**

Markus Hohenwarter, Florida Atlantic University, USA

Abstract

GeoGebra is a free, multi-platform tool that combines dynamic geometry, algebra, and calculus in one easy-to-use package for mathematics education in secondary schools. During this workshop I will show how GeoGebra can be used to foster discovery and experimental learning using multiple representations of mathematical objects. Special attention will be turned to interactive exercises and dynamic calculus examples. Participants will have the opportunity to explore hands-on activities with GeoGebra themselves. (see http://www.geogebra.org)

Title	**Introduction to Cabri 3D**

Kate Mackrell, Queen's University, Canada; and **Alison Parish,** Stowmarket High School, Suffolk, UK

Abstract

This workshop will introduce participants to using Cabri 3D to facilitate the learning of 2D and 3D school geometry and also as a tool for creating static and animated structures, such as truncating and folding polyhedra and cartoon characters who swing on a swing or skate on a Möbius strip. Techniques will range from quite simple, for those new to the software, to quite complex, for those with previous experience.

Title	**MathDesktop and the LTM Project**

Phil Ramsden, Imperial College London, UK; **Reinhard Simonovits** Bundeshandelsakademie Grazbachgasse, Graz, Austria; and **Bernd Thaller** Karl-Franzens University, Graz, Austria

Abstract

Universities and schools from eight European countries have been collaborating to develop Mathematica-based courseware together with didactical concepts for its use and material for in-service training. These resources, whose target group is secondary schools in Europe, enable teachers to pursue a problem-based and group-oriented approach to teaching mathematics that emphasizes methods and concepts over computations. Activities are designed to encourage an experimental, explorative approach.
The resources consist of extensions to M@th Desktop, developed by Reinhard Simonovits. M@th Desktop (MD) is an interactive teaching and learning software package for mathematics classes. It contains a collection of Mathematica notebooks with associated palettes and packages, and is designed to be especially compatible with a blended learning strategy, in which computer-based activities are used to complement other approaches. MD also provides tools for teachers and students to create their own palettes, notebooks and activities.
We propose a 90-minute workshop, to be led by Phil Ramsden, with M@th Desktop and LTM as its themes. The first 30 minutes will be presenter-led: colleagues will learn about the background to M@th Desktop and LTM; then, following a brief introduction to Mathematica, be introduced to the courseware and led on a tour of its main features.
The remainder of the session will be hands-on. Colleagues will mostly play the role of learners, undertaking some of the activities that the team have created, especially in the area of functions and graphs. Colleagues will have the opportunity to explore, and feed back on, the distinctive features of M@th Desktop, designed to make Mathematica's power accessible to students who are not necessarily Mathematica experts. Colleagues will also be invited to play the role of the teacher, and to reflect and feed back on possible modes of use of this courseware in the classroom, including the many opportunities that exist for customisation.

Title	***Mathematica* in Education**

János Karsai, University of Szeged, Hungary

Abstract

Mathematica, one of the leading CAS systems, is wordwide applied in the education at very different levels of very different fields. In the micro-course, the potential Mathematica users can get acquainted with the basic toolset and educational features. The participants will obtain a summary of the new features of version 6 and the directions of the further developments of Mathematica. The workshop gives the possibility to exchange the experiences, share the local developments, applications and teaching materials. Without any restriction of the topics, we mention the following topics:

- Formal and numerical calculations
- Handling singular cases
- Graphics and visualization
- Interactive, real-time applications
- Notebooks, typesetting, publishing with Mathematica
- Bugs, unusual work (like complex vs. real arithmetic)

Colleagues are kindly encouraged and asked to send problems, topics for discussion in theframe of the workshop.

Title	**Modelling with Sketchpad in the Teaching of Mathematics**

Homero Flores, Colegio de Ciencias y Humanidades, Universidad Nacional, México;
Cvetka Rojko, The National Education Institute, Slovenia

Abstract

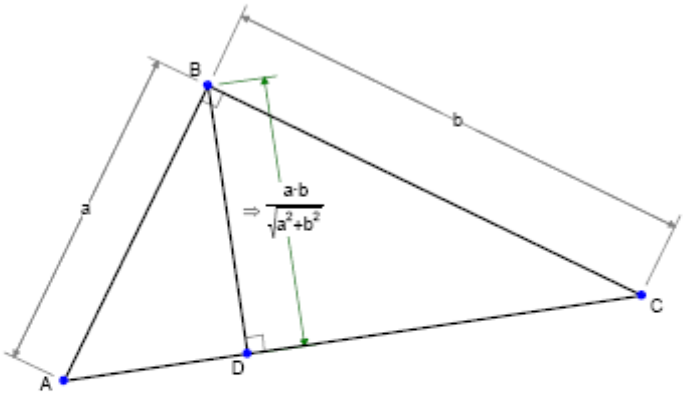
Modelling with Sketchpad in the Teaching of Mathematics Talking of education, people have now a-days got a strange opinion that every thing should be taught by lectures. Now, I cannot see that lectures can do so much good as reading the books from which the lectures are taken. I know nothing that can be best taught by lectures, except where experiments are to be shown. You may teach chymestry by lectures - You might teach making shoes by lectures! James Boswell: Life of Samuel Johnson, 1776 (Quoted in Galbraith, Blum, Booker, and Huntley (1998) Mathematical Modelling: Teaching and Assessment in a Technology-Rich World, Horwood, Publishing Limited) A model is a logical or mathematical structure used in science to account for a set of phenomena related to each other by certain relationships or a mathematical representation of a phenomenon, physical, economical, social, etcetera, that is made in order to better study and understand such phenomenon. And mathematical modelling is the process of getting a model. In mathematics teaching, a model could be a geometrical construction, a table, a graph or an algebraic expression, mainly a function. In a learner-centred teaching model the use of mathematical modelling has two goals. For the student the goal is to obtain a mathematical model that reproduces the data of the phenomenon to be modelled; and for the teacher the goal is that the student learns and uses the mathematics that lies behind the process of finding a model. Most of the time, the mathematics behind modelling is the mathematics we intend to teach our students. In this way, we face students with a practical situation where they must use their mathematical knowledge, communicate their findings to other students and the teacher, and, sometimes, convince them that what they found or their solutions are valid. In this workshop we illustrate the use of a DG software, The Geometer's Sketchpad, in a modelling activity. The goal is to reflect on the use of Dynamic Geometry to connect Algebra with Geometry at secondary level (15-17 year old learners) through the solving of a typical optimization problem. It is intended for teachers and interested attendees with or without Sketchpad experience.

Title	**Solving Problems with Geometry Expressions and CAS**

Philipp Todd, Saltire Software, Oregon, USA

Abstract

Geometry Expressions (www.geometryexpressions.com) is a computer application th unlike other interactive geometry systems, can automatically generate algebraic expressions from geometric figures. For example, in the diagram below, the user has specified that the triangle is right and has short sides length a and b. The system has calculated an expression for the length of the altitude:



Though producing symbolic output from symbolic input, the output is limited things o fairly direct, geometrical nature. For example, Geometry Expressions can produce a symbolic expression for some area but not compute an optimum for that area over som symbolic parameter; it can produce parametric, and often implicit equations for a trace curve but not perform the analysis necessary to determine the singularities of this curv For such further analysis the symbolic output from Geometry Expressions may be pass to a CAS. Geometry Expressions can thus be viewed as a bridge between the worlds o Dynamic Geometry and CAS.

In this workshop, participants will use Geometry Expressions and CAS in combinatio solve mathematical problems. They will learn effective techniques for formulating problems in Geometry Expressions, in such a way that the results are amenable to analysis in CAS. They will be exposed to the use of Symbolic Geometry to set up a problem in algebra or calculus, and the use of CAS to finish off a problem in geometry

Title: Using Computer Algebra / Technology for modelling and application

Burkhard Alpers, University of Aalen, Germany

Abstract

Aim:
It is the aim of this workshop to shed more light on the role technology, in particular Computer Algebra Systems, can play in mathematical modelling and application projects that are part of the mathematical education.
Content:
The organizer will first give an exemplary introduction into the kind of models and modelling activities that might be of interest in mathematical education. Then, participants will choose from two small modelling tasks (e.g. windscreen wiper) and try to answer an application question using the CAS Maple (support will be provided for those who are not familiar with Maple). In the third part, the participants will discuss the role the CAS played in working on the modelling tasks and try to identify general patterns of usefulness as well as parts where a CAS (or technology in general) cannot help.

Title	**Visualization of solid figures in practice with Euler3d and MAPLE**

Lajos Szilassi and László Kóródi, University of Szeged, Hungary

Abstract

Aim: The aim of this workshop is to give a brief introduction into the usage of Euler3d and its connection with MAPLE.

Content: "Euler3d" is a free software designed to display solids in 3-dimensional space. It is currently available for the Windows operating system and can be freely downloaded from http://www.euler3d.hu/index.php?lang=EN.

The polyhedron (or any other kind of solid figure) to be displayed has to be described by the numerical coordinates of its vertices. The faces are given by the ordered vertices belonging to the face. When a face is registered, the program executes certain verifications. (The contributed talk of the organizers will deal with the background of these verifications.)
On the workshop we will make the participants acquainted with further possibilities of Euler3D, for example moving of the figures and the camera, use of colors and layers, transformations, animations.
The software is further able to determine the given polyhedron's dual with respect to any base sphere except if the center of the sphere lies on a plane determined by one of the faces.
It is also possible to import objects described in the VRML1 format. In particular, the software is capable of displaying surfaces produced by MAPLE.

Title	**WebMathematics Interactive 2**

Zoltán Kovács, Bolyai Institute, University of Szeged, Hungary

Abstract

WebMathematics Interactive (WMI) is an open source mathematics software. It mainly supports solving problems and exercises in mathematics for the age of 14-20. WMI also helps in verifying solutions and discovering skills and level of knowledge. Users need only internet connection and a web browser to use WMI.

Nowadays ever more people study mathematics in universities or colleges. However, study groups are usually too big for the teacher to help each student to gain a deep understanding of the educational material. The spread of computers and the internet provides a reliable aid for many more students as an "every day tool". WMI also supports practicing typical exercises and preparation for the exams.

The WMI program is written in PHP and runs on an arbitrary Linux server. Many other software are also contained in the underlying software packages, like LaTeX, latex2html, Apache, PostgreSQL, xeukleides and so on. The software is primarily developed in the University of Szeged, Bolyai Institute. The project was supported by the Ministry of Informatics and Communications, Hungary, in 2003. The most recent version can be downloaded from the Sourceforge server, however it can be used immediately as well on `http://wmi.sf.net`.

During the CADGME 2007 workshop the audience gets a closer look to the newest version of WMI called "WMI2". This new generation web application supports web 2.0 technologies to ease user interaction and enhance abilities for third party developers. The audience is planned to gain practical experience in developing didactical material using WMI2 which implies that the productive work should not be more than a few minutes after taking part in the workshop.

Title	**WIRIS, mathematics for education**

Ramon Eixarch, Maths for More, Barcelona, Spain

Abstract

WIRIS, mathematics for education

The workshop will be composed of a **brief presentation** and a **hands-on session** of WIRIS tools for mathematics education.

WIRIS is a family of products that offers solutions including mathematical calculations, experimental laboratories and self evaluation questions for High School education but also university first year courses. www.wiris.com

Our tools are available in English, French, Spanish, Italian, Dutch, Estonian and Catalan. If your language is not yet listed it will be included in that list. German is coming very soon ☺

WIRIS CAS = CAS + DGS + on-line + Free access

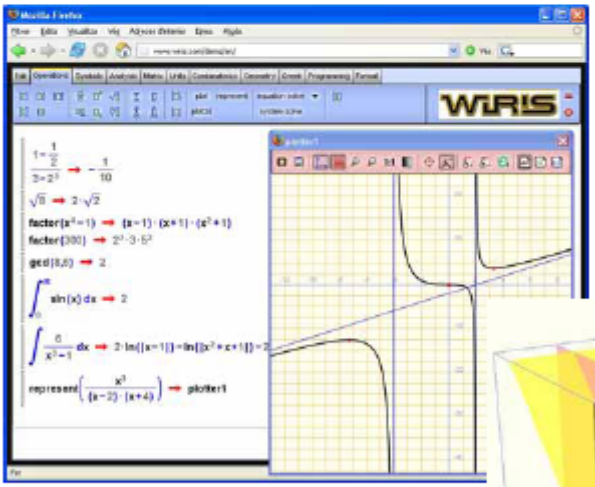
WIRIS CAS is an on-line platform for mathematical calculations designed for education. It is in fact a CAS that also includes a DGS inside. You can access a powerful calculation toolbar through an HTML page that includes integrals and limits calculation, function graphing in 2D or 3D or symbolic matrices manipulation. It covers all mathematical topics from High School to University level (Calculus, Algebra,...).

WIRIS is a free access in several European educational portals. Those versions are sponsored by the corresponding Ministry of Education and are free to use for teachers and students.

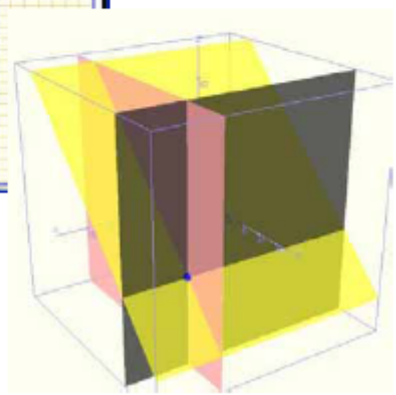
Visit complete demonstration versions at

ENGLISH	www.wiris.com/demo/en/	SPANISH	www.wiris.com/demo/es/
FRENCH	www.wiris.com/demo/fr/	ITALIAN	www.wiris.com/demo/it/

WIRIS CAS basic functions



Graphics three planes and intersection point in 3D



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Working Groups

Title	**Automated Reasoning and Mathematical Education**

Bruno Buchberger, RISC, University of Linz, Austria

Abstract

Mathematics is the science that produces insight and solves problems by reasoning. In parallel to the historic development of mathematics, in mathematical education mathematical reasoning is evolving gradually: At the beginning of mathematical education the properties of mathematical objects like numbers and geometrical objects are studied by experimentation and observation. Only in a more mature age, the question of "why" certain observations probably hold in "all" possible situations moves into the foreground and is answered by giving a finite chain of arguments, i.e. a "proof". The technique of arguing, reasoning, proving is then evolving to its full potential in university mathematics. This technique is now understood as a process whose guiding rules can be explicitly and comprehensively described in a completely mechanical (algorithmic) way. In fact, the automation of reasoning (computer logic) has seen enormous theoretical and practical progress in the past four decades and by now constitutes an important part of symbolic computation with computer algebra being the other important constituent. In fact (see the editorial of the Journal of Symbolic Computation), computer algebra and computer logic are intimately interrelated and their important interactions and synergies are studied in the recent literature.
While computer algebra is now more and more used in mathematical education, automated reasoning techniques and systems so far have hardly been introduced in education. In the session "Automated Reasoning and Mathematical Education" of CADGME, we will focus on

- automated reasoning systems that offer specific support for use in high-school and academic mathematical education,
- the role of reasoning in mathematics and the importance of proof training,
- possibilities of supporting logic training and proof training by recent symbolic computation systems,
- the interaction of (computer-supported) experimentation and (computer-supported) reasoning in mathematical exploration,
- case studies and reports on classroom experiments on formal (computer-supported) reasoning in the frame of mathematical education,
- special reasoning tools for special mathematical theories (e.g. geometry, elementary arithmetic, set theory, elementary analysis etc.) and their use in mathematical education.

Submission of papers to the Session on "Automated Reasoning and Mathematical Education" should proceed via the ordinary submission procedure of CADGME but should be marked by "Session ARME".

Title	**Computer-aided Experiments and Visualization in Education**

János Karsai, University of Szeged, Hungary

Abstract

Experimentation and visualization are organic parts of scientific research and education. In Mathematics, such methods have been used mostly in preliminary examples or to illustrate theories since they do not have the power of proof in the rigorous mathematical theories. But thanking to the extensive usage of high powered desktop (or even laptop) computers with extreme graphical capabilities in the mathematical research, experimental and visualization methods are more and more important. With the help of computing tools, difficult and complicated algorithms and constructions (numerical, geometrical, etc…) can be performed, as well as they make possible to introduce new methods, which had not been possible before. In addition, experimental study proved to be quite effective in studying mathematical problems. The new

features have been leading to the fast development of constructive methods, and several classical fields. Even more, new fields born and have become very important.
Experimental and visualization methods and tools are also highly involved in the education, where either manual or computer-aided experiments can help a deeper and more comprehensive knowledge. In the session "Computer-aided Experiments and Visualization in Education" of CADGME, we will focus on the computer-aided experimental and visualization methods in:

- problem solving
- understanding mathematical theories
- applications in sciences and engineering

Hence either didactic or research submissions are welcome concerning

- visualization of mathematical objects and methods
- constructions
- the role of experimentation in mathematical exploration
- applications for classroom or individual study
- special applications for mathematical theories (e.g. geometry, algebra, analysis, numerical methods, statistics etc.) and their usage in the education
- applications in special fields of sciences, engineering, informatics etc.

Submission of papers to the Session on "Computer-aided Experiments and Visualization in Education" should proceed via the ordinary submission procedure of CADGME but should be marked by "Session CEVME".

Title	**Future Trends in Interactive Geometry**

Ulrich Kortenkamp, Pädagogische Hochschule Schwäbisch Gmünd, Germany

Abstract

Interactive (or Dynamic) Geometry software (DGS) is an accepted tool for teaching mathematics now. Those who are accepting the use of computers in schools are well aware of the potential of these software packages, and there exist lots of resources, pre-designed activities, explorations, illustrations, and exercises that can be used off-the-shelf to exploit a great part of this potential. However, in recent years several new trends and developments have emerged: some packages allow for a tighter integration with algebraic and symbolic approaches, some offer novel user interfaces, others include methods for numeric simulation, etc. As these developments are unfamiliar to most users (teachers and scientists), there is no or little formal evidence of their usefulness or evaluation results. This session tries to give an overview over both innovative approaches to DGS and innovative use of traditional DGS, and to demonstrate how these could be applied to mathematical education. We invite submissions on the subject of

- Innovative features of DGS,
- Specialized and experimental DGS,
- Interaction between DGS and other mathematical software
- Interaction between DGS and the real world,
- New mathematical methods that are used to improve DGS,
- Non-standard classroom activities that make use of new features or make creative use of "old features",
- Evaluation results for DGS use that could be used to identify new directions in the development of DGS,
- Usability studies for DGS,
- and other topics that are related to the above.

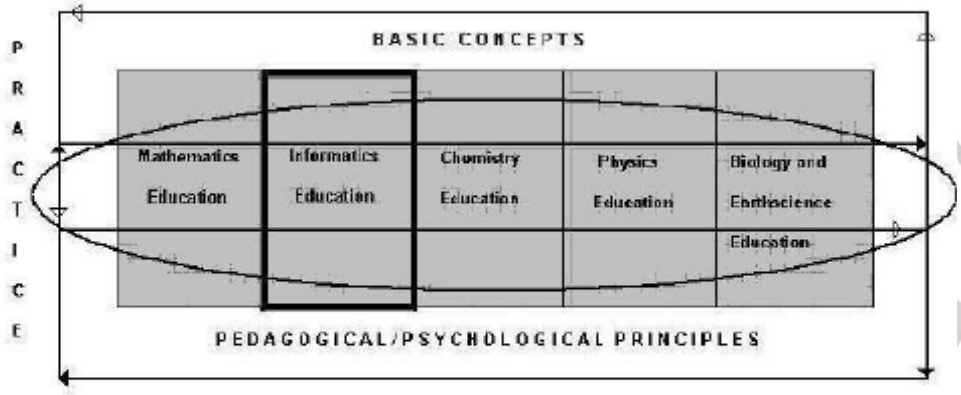
Submission of papers to the Session on " Future Trends in Interactive Geometry " should proceed via the ordinary submission procedure of CADGME but should be marked by "Session FTIG".

Title	**Informatical Concepts and CAS**

Karl Josef Fuchs, University of Salzburg, Austria

Abstract

Topic: Informatical Concepts and CAS
The topic must be seen as an invitation both to mathematics AND computerscience educators. The starting point will be the following published Model of Computer Science Education (MCSE).



(Fuchs, Karl Josef (2005): Published in: *Innovative Concepts for Teaching Informatics, ISSEP 2005, S. 9*)

Starting form this concept the significance of CAS in regard to programming which is equal to modelling in a modern concept of programming should be discussed. Hence reports on diverse experiences in using CAS for programming (as the debatable role of programming in teaching and learning of CAS or the different paradigms and approaches) will be welcomed to find out a appropriate position of CAS in an MCSE.

Title	**Integrated Use of Tool in Mathematics Education**

Eva Vasarhelyi, University of Salzburg, Austria

Abstract

The main topic of this working group of the combination of

- the tools for manual visualisation - Lenart Sphere,
- calculators - ClassPad and computer
- computer animations - Cabri 3D.

Title	**Relating procedural and conceptual knowledge of mathematics through CAL**

Djordje Kadijevich, Megatrend University and Mathematical Institute SANU, Belgrade, Serbia

Abstract

Although developing and relating procedural and conceptual mathematical knowledge is a very important goal of mathematics education, it is rarely attained, especially when their relation is in focus. However, few CAL studies have examined whether their technology-based treatments search for and/or make use of possible links between procedural and conceptual mathematical knowledge. Main discussions within this WG will be directed towards but not limited to the following issues:

1. Frameworks that explains how procedural and conceptual knowledge may be related in technology-supported learning environments;
2. Desirable features of learning, teaching or software (CAS, DGS, etc.) that would enable relating procedural and conceptual knowledge within computer-based learning environments;
3. Implemented technology-supported learning environments that enabled relating procedural and conceptual knowledge;
4. Limitations of present software (CAS, DGS, etc.) concerning the promotion of the links between procedural and conceptual knowledge;
5. The impact of different technologies (i.e. learning opportunities of different technologies) on establishing links between procedural and conceptual knowledge.

To prepare for their participation in the activities of this WG, colleagues interested in this topic may first examine Chapter 3 of Kadijevich (2004).

References

Haapasalo, L. & Kadijevich, Dj. (2000). Two types of mathematical knowledge and their relation. Journal für Mathematik-Didaktik, 21, 2, 139-157.

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Kadijevich, Dj. (2004). Improving mathematics education: neglected topics and further research directions (doctoral dissertation). University of Joensuu: Publications in Education (No. 101). Available at www.joensuu.fi/research/index.html

Title: Research perspectives of the impact of dynamic mathematics on teaching and learning

Stephen Hegedus and Luis Moreno, University of Massachusetts, Dartmouth, USA

Abstract

Over the past 20 years, software in mathematics education has evolved in a variety of directions. Increasing computational power, fine resolution screens and interactive devices that allow developers to create rich, visual, representations that enable students better access to complex, and abstract mathematical ideas. As a result, we have a major innovation: *dynamic mathematics*. Now we need to move towards a new ecology, a new environment, to develop the ideas that should guide 21st century education.

Our working group focuses on the development of theoretical perspectives on the design, use and impact of new technologies such as dynamic geometry around certain themes. Mathematics is a transforming force in society. All processes of **dissemination** of mathematical knowledge must be opened. Mathematical Knowledge has been growing, extending, developing, organizing, compressing, compacting and these processes impact human cognition whose nature is essentially mediated. We adopt the viewpoint of cognition as *mind dynamics* that transform pieces of knowledge into understanding. Mind extends beyond the skin into social space.

In a connected classroom, technology is not only a prosthetic device (allowing the teacher to enhance her/his practice) but an instrument to *transform the communication as well.* Knowledge emerges mainly as a communal, distributed product thus we must look for the seeds of dissemination. We encourage

contributions that range from emerging ideas to established bodies of research to address our primary focus of theorizing what happens in a classroom that uses dynamic mathematics technology.

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